

The University of Chicago

A STUDY OF NUMBER ABILITY IN
CHILDREN OF AGES THREE TO SIX

A PART OF A DISSERTATION SUBMITTED TO
THE FACULTY OF THE DIVISION OF THE
SOCIAL SCIENCES IN CANDIDACY FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF EDUCATION

1932

By

KATHERINE LOUISE McLAUGHLIN

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CHAPTER I

INTRODUCTION: PURPOSE, VALUES, AND LIMITATIONS OF THE INVESTIGATION

The scientific study of number development has been carried on during the past two decades more vigorously and more extensively than at any other previous time. A recent survey of the field discloses, "In 1900 there were but thirty published studies of arithmetic which could claim to be quantitative and scientific; in 1929 there were approximately five hundred such studies in print . . ."¹ Three hundred ninety-eight of these, published during the years 1913 to 1930, have been critically analyzed with a view to showing dominant tendencies.² One investigator, interpreting recent trends in relation to earlier developments, summarizes the situation as follows:

"Arithmetic experienced a great period of overexpansion from the time of Warren Colburn's influence down to 1900. The first problem to receive the scientific attention of the student of education was the determination of the means of reducing this overexpansion and the elimination of useless subject matter. This led at once to studies of a more positive character to discover some fundamental principle upon which the construction of the curriculum might be based. This search for a fundamental principle has stimulated two main types of research: first, investigations based upon the principle of social utility or social demand, and second, investigations based upon a psychological consideration of the nature of the number system and the difficulties which

¹G. T. Buswell, A Critical Survey of Previous Research in Arithmetic, 29th Yearbook, National Society for the Study of Education, p. 470. Bloomington, Illinois: Public School Publishing Co., 1930.

²G. T. Buswell and C. H. Judd, Summary of Educational Investigations Relating to Arithmetic, Supplementary Educational Monographs, No. 27 (1925), and Supplements No. 1-5 (1925-29). Chicago: Department of Education, University of Chicago.

children encounter in mastering it."¹

The present study is an investigation of the second type referred to in the preceding quotation. It deals with the beginnings of number experiences in preschool children. The recent and rapid growth of preschool education has shown the need of a more thorough understanding of early mental development for purposes of effective training and guidance at early ages.

Gesell in evaluating the psychological significance of the preschool age states,

"From the psychological and educational standpoint the conditions of the preschool period are interesting and challenging. . . . The tacit assumption is that it is not an educational period at all. Psychologically nothing could be more erroneous than such an assumption. In a certain sense the amount of mental growth which takes place in the first sexennium of life far exceeds anything which the child achieves in any subsequent period. Indeed, it may be doubted whether all his scholastic strides taken together bulk as much as his brilliant advance from the stage of protoplasmic vegetation at birth to the mastery of physical and personal relations--languages, art, and science--which he has attained when he first slings his school bag over his shoulder."²

Problem of the Investigation.-- The purpose of the investigation to be reported in the following pages is twofold. It seeks (1) to analyze in preschool children the development of three phases of experience, namely, counting, recognition of number aggregates, and combination of aggregates; and (2) to determine to what extent the development of ability to deal with number is conditioned by general intelligence. The problem was studied by means of two sets of tests administered to all subjects; one a test of ability to use number and the other a test of general intelligence.

Values.-- The investigation yields certain practical values:

(1) It offers to students of mental development, particularly to teachers and parents of young children, objectively derived inventories of abilities to deal with number ideas at early age levels. Such inventories help to supply a scientific basis³

¹G. T. Buswell, op. cit., p. 448.

²Arnold Gesell, The Preschool Child, p. 7. Boston: Houghton Mifflin Co., 1923.

³R. L. West, C. E. Green, and W. A. Brownell, The Arithmetic Curriculum, 29th Yearbook, National Society for the Study of Education, p. 108 ff. Bloomington, Illinois: Public School Publishing Co., 1930. Points out need for scientific study of

for teaching and learning number in grades one and two. A survey of teaching procedures,¹ current textbooks,² and courses of study³ for primary grades indicates the wide discrepancy in school practices due to lack of objective data regarding children's abilities in the field of number.

(2) It presents an analysis of different methods employed by children prior to school instruction in dealing with the processes of counting, recognition of aggregates, and combination of aggregates. For example, in combining 5 and 4, the correct answer may be reached by several methods—counting from 1 to 9, combining 5 and 5 and subtracting 1, counting-on by 1's or by 2's from 5, or finally 'seeing' the correct sum without preliminary calculations. Analysis of the varying degrees of maturity indicated by such

learning steps in various processes as an aid in establishing a more adequate basis for curriculum construction.

¹S. C. Parker, Types of Teaching and Learning, p. 148. Boston: Ginn & Co., 1923. Reports considerable disagreement concerning age at which study of number should begin, in some instances simple counting is prohibited in first grade.

²B. R. Buckingham and J. MacLatchey, Number Abilities of Children When They Enter Grade One, pp. 473-74, 29th Yearbook, National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Co., 1930. "The conventional arithmetic textbook, a book intended to be placed in the hands of third grade children begins with the presentation of a gaudy picture in which certain priggish children, impossibly clean in appearance, are occupied with hoops, marbles, dolls, or whatnot of the apparatus of play. . . . The author tells the children to look at the picture. 'How many girls are there? How many boys are there? How many girls are sitting down? How many boys are sitting down? How many boys are standing up?' . . . Within half a dozen pages . . . the child is called upon to solve problems involving the adding of one digit numbers and to complete abstract number facts such as four and eight . . . Within the space of twenty pages the child has apparently progressed from telling how many girls there are in the picture to the solving of a problem such as, 'Harry was 4 years old when his sister Julia went to school. Julia has been in school eight years. How old is Harry?' . . . [it] can be right at only one end or the other of the material in the first score of pages."

³R. I. West, C. E. Green, and W. A. Brownell, op. cit., pp. 108-17. Presents wide divergences in time allotment, content and outcomes in an analysis of six courses of study recently published.

methods 'points the way' to necessary instruction.¹

(3) It indicates the types of difficulty encountered in number learning at early stages. Other investigators report that early difficulties in number learning persist and become causal factors of retardation in arithmetic.² Schmitt, drawing inference from a study of thirty-four children, "extremely retarded in arithmetic but not feeble-minded," says, "achievement in arithmetic is dependent upon a hierarchy of habits which depend upon each other in sequence. If a hiatus . . . occurs at any essential point confusion results for the child."³

(4) It presents the methods of generalization developed by preschool children in their attempts to deal with number. The dependence of number ideas upon ability in generalization and abstract thinking has been emphasized by Royce,⁴ Stern,⁵ Judd,⁶ and

¹G. T. Buswell, op. cit., p. 453. "Curriculum construction has apparently come to a crossroad. A more adequate technique needs to be discovered. Among the various suggestions which are encountered, two seem to be promising. One of these is that a good deal of light might be shed on the problem by more detailed analyses of pupils' reactions in dealing with quantitative situations, letting the difficulties which these exhibit point the way to the necessary instruction."

²G. T. Buswell, with cooperation of L. John, Diagnostic Studies in Arithmetic, Chap. I. Supplementary Educational Monographs, No. 30. Chicago: Department of Education, University of Chicago, 1926.

J. S. Orleans and F. E. Seymore, Arithmetic in Rural and Village Schools of the State of New York. Albany: University of State of New York Press, 1926; Bulletin No. 865 (December, 1926), University of State of New York. Both studies describe difficulties in learning fundamental processes.

³Clara Schmitt, "Extreme Retardation in Arithmetic," Elementary School Journal, XXI (March, 1921), 529-47.

⁴Josiah Royce, Supplementary Essay: The One, the Many, and the Infinite, First Series Gifford Lectures, p. 529. New York: Macmillan Co., 1901.

⁵William Stern, Psychology of Early Childhood (Trans. from 3rd Edition, Anna Barwell). New York: Henry Holt & Co., 1924. Pp. 558.

⁶Charles H. Judd, Psychological Analysis of the Fundamentals of Arithmetic. Supplementary Educational Monographs, No. 32. Chicago: Department of Education, University of Chicago, 1927.

others in studies dealing with subjects beyond the preschool age. The value of objective data regarding methods of generalization at earlier ages is obvious.

(5) It shows the correlation between ability to deal with number and general intelligence.

(6) It refines one type of test which should be used in determining mental maturity of first grade entrants--thus giving, for purposes of ability grouping and differentiated instruction, a more rational basis than is afforded by the 'chronological' one now in general use.¹ That school entrants differ in readiness for the work of the first grade is common knowledge among teachers of beginners² who point out that the high percentage of non-promotions in the first as compared with other grades is largely due to this fact.³

Limitations.— Certain limitations are inherent in the investigation:

(1) Formal testing of preschool children creates an artificial atmosphere. To offset this limitation a natural situation had to be maintained. Tests, therefore, were presented in the form of games in order to approximate as closely as possible the play

¹Margaret Holmes, "Investigation of Reading Readiness of First Grade Entrants," Childhood Education, III (January, 1927), 215-21. Shows general practice as laid down by law regulates school entrance on a basis of chronological age.

²Report of Research Committee, p. 4 ff. and p. 32. Sacramento: California Kindergarten Primary Association, 1928. A study of 8,921 first grade entrants reports 19 per cent non-promoted at the end of first semester. Among causes, listed mental immaturity received highest frequency rating—"not necessarily low intelligence, but simply mental age too young to acquire reading skills."

³Walter P. Percival, Study of Causes and Subjects of School Failure. Dissertation, Teachers College, Columbia University, 1926. "Grade One is the greatest failing grade being responsible for 30.2 per cent of the failures in the cities and 24.1 per cent of all failures." The latter includes all failures in rural and urban grades.

K. L. McLaughlin, "Providing for Immature School Entrants," Childhood Education, VII (October, 1930), 85-91; cf. Third Biennial Report, California Kindergarten Primary Association (1929), pp. 49-54. Reports that ability grouping and differentiated instruction are factors in reduction of non-promotions in eight cities of California.

interests of the subjects.

(2) The plan for the testing program necessitated different subject groups for the successive ages. The justification offered for comparing different persons is that attainment at successive ages for different groups approximates the attainment which the same group of children would make in successive years. Conditions of scientific exactness would, doubtless, be met more adequately if measures obtained were the results of annual re-tests of the same instead of different subject groups. This limitation must be taken into account in all inferences and conclusions.

(3) Since all subjects were enrolled in nursery schools or kindergartens they may represent a more selected group than children whose home situation precludes such experience. The necessity of a routine time and place schedule for testing obviated inclusion of non-school subjects as these were not readily accessible. This selective factor adds a further limitation.

(4) The large number of responses required by the test (approximately 100), the time required (an average of twenty-five minutes per child tested), and probable inadequacies within the test may be sources of difficulty. This possibility is kept in mind in interpreting results.

(5) Other possible causes of fluctuations in obtained measures are the general emotional reactions of subjects, their general health conditions, and interest appeal or lack of it. Although these are possible limitations, meticulous care has been exercised to avoid as far as possible errors due to their presence.

CHAPTER II

EXPERIMENTAL PROCEDURES

Preliminary testing: Class One.- The study herein reported was begun in the summer of 1927. At that time several tests in counting and combining numbers were tried on children from the Chicago Orphan Asylum. Subsequently the tests were modified and extended to cover recognition of group ideas, that is, cardinal number. The preliminary tests were administered to thirty-five children ranging in age from twenty-seven to eighty-one months; eight were enrolled in the nursery school or the kindergarten of the University Elementary School, University of California at Los Angeles; seventeen, in resident groups of the Chicago Home for Jewish Children and the Chicago Orphan Asylum; and ten, on the accepted list of kindergarten entrants of the University Elementary School, University of Chicago. (During the summer of 1928, a rudimentary form of the test was administered to a group of Oriental and Hawaiian children in Honolulu.) All children included in this first stage of the study are referred to throughout this report as belonging to Class One. Data obtained from Class One are not employed in the analyses of this and the following chapter, but in Chapter IV reference is made to responses of two year old children.

Experimental groups: Classes Two and Three.- A second group of children, designated in this report as Class Two, includes one hundred children between the ages of four and six; eighty-four were enrolled in public and sixteen in private kindergartens and nursery schools of southern California. Table I gives the distribution of this class by chronological ages and by schools.

School 1, the kindergarten of the Roosevelt Public School, Santa Monica, California, is characterized by the Advising Director of Research, Santa Monica City Schools, as follows:

"The Roosevelt School, in which this kindergarten is located, is a typical Santa Monica public school, and is probably as representative of families originating in various parts of the United States as any school could be. There is no doubt, in the opinion of members of the research staff of the Santa Monica City Schools, that what would be true of the mental

TABLE I

DISTRIBUTION OF CHILDREN IN CLASS TWO BY AGES AND SCHOOLS

Age by months	School Attended						Total
	1	2	3	4	5	6	
48-53 months	-	3	3	12	5	2	25
54-59 months	12	8	-	2	-	3	25
60-65 months	20	2	-	2	-	1	25
66-71 months	16	3	-	1	-	5	25
Total	48	16	3	17	5	11	100

processes of these children would be equally true of kindergarten children in general."¹

School 2, the kindergarten of the Alexandria Avenue School, Los Angeles City School System, is the Demonstration School of the University of California at Los Angeles. It is a typical twenty-teacher public school located in the midst of a rapidly developing residential section. The economic and social conditions approximate those of an average American community.

Schools 3 and 4 are the nursery school and the kindergarten respectively in the University Elementary School, University of California at Los Angeles. The Director of the Training Department says, "The children in these two classes are all of American parentage and of rather better than average social status."²

Schools 5 and 6 are the nursery school and kindergarten of the Progressive School, Hollywood, California, a private school patronized by professional and business classes of the community. Many fathers and several mothers were associated with the motion-picture industry as directors, artists, or writers; others were physicians, attorneys, or business people of Hollywood.

A third group of children, designated in this report as Class Three, included twenty-five subjects ranging in age from three to four years. Twelve of these attended the nursery school

¹Dr. J. Harold Williams, Professor of Education, University of California at Los Angeles, and Advising Director of Research, Santa Monica City Schools.

²Dr. Charles W. Waddell, Director of Training and Professor of Education, University of California at Los Angeles.

afore mentioned and thirteen others were enrolled in the nursery school of the Division of Psychology and Educational Research, Los Angeles City School System. Mothers of the latter subjects were employed in gainful occupations and their economic status varied from that of slightly below level of self-support to that of financial independence.

Selection and Attitude of Subjects

Chronological age at the time of the administration of the number-test together with teachers' estimate of intelligence gave the basis for selection of subjects. Effort was made to select bright, dull, and average subjects in such proportion as to yield balanced groups for the three successive years. No child known to be subnormal or pathological was included. The school records were used in determining ages, except in cases of doubt and then the family records were used. Fractions of a month less than fifteen days were discarded; all over this amount were credited as one full month. Ages were carefully considered in order to secure a balanced group for the first and second six-month period within each successive year. For example, ages of the first twelve children of Class Three fell in the first half of the three to four year period and the remainder in the second half. The same plan was followed in selecting subjects for Class Two, the four-five and the five-six year groups.

The attitude of the children toward the tests was consistently a play-attitude. They enjoyed manipulating the materials and the boxes in which they were kept. Some, Pandora-like in their curiosity, peeped under the lids to discover the contents. A colored picture of Santa Claus on top of one box never failed to attract attention and at times to call forth a breathless narrative of "what Santa Claus brought me for Christmas." Many insisted on kissing the picture as a mark of affection. Not one of the subjects evidenced a realization that the "games" were tests.

The attitude toward the examiner was friendly and cooperative. After the first visit or two to the school her reappearance was marked by a cordial welcome that not infrequently included an insistent invitation to "join in" their play, or in the birthday party for Jane or Bobby. That this attitude was carried over into the testing period was clearly evidenced by ingenuous and intimate accounts of home and school affairs that revealed absence of restraint on the part of the youthful narrator. In one or two instances when there was some hesitancy about playing the "games" a plausible excuse was found to delay them until another time. Usually the timid were heartened by a cheering compliment, or an

interested inquiry about some trivial possession or recent accomplishment, or by other germane attentions that the intimate situation seemed to call forth and vitalize for the child.

General Plan for Testing

Arrangements for carrying out the testing program of the investigation were made some time in advance with administrative and supervisory officers of the several schools. Following this arrangement a conference was held with the teacher of the group to be tested, the nature of the experiment was explained and a tentative list of children, selected on the basis of criteria as described in the foregoing section, was agreed upon. With the cooperation of the principal or supervisor a time schedule was worked out and a suitable room for the testing assigned.

With few exceptions all the children tested were enrolled in morning sessions. This fact restricted the time of testing to the forenoons. Furthermore, the examiner who administered the number tests was available only on Tuesday and Thursday morning. The tests were scheduled, therefore, from eight forty-five to eleven forty-five on these two mornings each week throughout the school year 1929-30, beginning on October 3. All tests were given by the same examiner except those given to five in Class Three. These were administered by a graduate student who has specialized in nursery school tests and measurements.¹

The average time required for the test was twenty-five minutes. One extremely negative child required forty-five minutes. In spite of the lengthy period for immature subjects, fatigue was a negligible factor, possibly because the tasks in the tests were novel and varied. The children not infrequently evinced a too active interest (especially if the examiner delayed in making notations on the response record), in a "now what-next-" attitude of expectancy. Some even lingered after completion of the test to play with the materials. Such irregularities were ordinarily anticipated and prevented by the impressive pinning of a small flag as an award on the youngster's frock or coat. With this he bounded off in genial mood to show his playmates "what-I got."

The place for testing was usually a small room in close proximity to the classroom. On one occasion an upstairs room, used temporarily, was abandoned when it was discovered that test responses of one or two children were conditioned by a fear reaction

¹Miss Elizabeth Pell, M.A. (1930), University of California, Berkeley, California.

aroused in climbing a long flight of stairs.

The necessary equipment included a low table and a couple of kindergarten chairs of standard heights. A dark oilcloth covering for the table made the light colored materials of the several tests show to advantage.

Measures of Mental Maturity and Teachers' Ratings

All children examined were given an individual intelligence test. Sixty-four were given the Stanford-Revision of the Binet-Simon Test; forty-eight, the Detroit Kindergarten Test; and thirteen, the Kuhlmann-Binet. Scores on the latter two tests were converted into equivalent I.Q. scores on the Stanford-Binet Scale. The number and per cent tested by the three measures are as follows:

	<u>Intelligence Test</u>	<u>Number</u>	<u>Per Cent</u>
Class Two	Stanford-Revision Binet-Simon .	52	41.6
	Detroit Kindergarten Test . . .	48	38.4
Class Three	Stanford-Revision Binet-Simon .	12	9.6
	Kuhlmann-Binet	<u>13</u>	<u>10.4</u>
	Total	125	100.0

The tests were administered under the supervision of an instructor from the Department of Psychology, University of California at Los Angeles,¹ and by staff members of the research divisions of the cooperating schools.² Mental ages and I.Q.'s for Class Two appear in Tables II and III and those for Class Three in Table IV.

In order to secure supplementary data for diagnostic purposes, specially prepared report-forms were submitted to the several teachers for information on the following: number interest of the children, type of temperament, family membership, economic condition of family, general health, and other significant factors conditioning development.³ Teachers' ratings of number interest appear in column 5 of Tables II-IV. Other pertinent data are

¹Dr. E. B. Sullivan, Assistant Professor of Psychology, University of California at Los Angeles.

²Elizabeth Bruene, School Counselor, and Marion Wallace, Training Teacher, Nursery School, University Elementary School, University of California at Los Angeles; Miss Clarissa Bacon, Research Staff, Santa Monica City Schools; Miss Annetta Nix, Division of Psychology and Research, Los Angeles City School System.

³These records are summarized in Appendix, pp. 151-57.

TABLE II

DATA FOR ANALYSIS IN NUMBER ABILITY OF FIVE-YEAR AGE-GROUP

Subjects	Sex	C.A.	M.A.	I.Q.	T.R. (number interest)	Number-Test Score		
						First Trial	Second Trial	Average
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	G	5 - 11	5 - 6	93	C	51.6	49.6	50.6
2	G	5 - 11	7 - 0	118	B	68.8	61.8	65.3
3	G	5 - 10	8 - 0	138	A	119.0	122.0	120.5
4	B	5 - 10	7 - 1	122	A	125.0	138.0	131.5
5	B	5 - 10	6 - 7	113	A	59.4	49.4	54.4
6	G	5 - 9	5 - 9	100	B	60.4	63.4	61.9
7	G	5 - 9	6 - 8	113	B	60.6	62.6	61.6
8	B	5 - 8	6 - 8	118	A	101.8	100.9	101.3
9	B	5 - 8	5 - 0	88	C	29.9	29.8	29.8
10	B	5 - 8	5 - 0	88	C	35.6	37.6	36.6
11	B	5 - 8	5 - 1	90	B	52.4	58.4	55.4
12	G	5 - 8	6 - 2	109	A	90.5	99.0	94.7
13	G	5 - 8	6 - 9	119	A	100.6	101.7	101.1
14	G	5 - 8	6 - 10	121	A	105.0	106.0	105.5
15	G	5 - 7	8 - 5	115	C	46.4	58.7	52.5
16	B	5 - 7	5 - 11	106	C	21.3	23.3	22.3
17	G	5 - 7	5 - 11	106	B	118.9	117.9	118.4
18	B	5 - 7	7 - 4	132	A	89.5	90.8	90.1
19	G	5 - 6	5 - 0	91	B	66.6	64.6	65.6
20	G	5 - 6	6 - 2	112	A	74.6	115.9	95.2
21	B	5 - 6	4 - 11	89	C	66.9	76.8	71.8
22	G	5 - 6	7 - 3	137	B	30.6	31.2	30.9
23	B	5 - 6	7 - 2	130	B	49.1	47.3	48.2
24	G	5 - 6	5 - 5	98	B	34.7	40.7	37.7
25	G	5 - 6	4 - 8	82	A	30.2	28.2	29.2
26	B	5 - 5	6 - 6	120	C	53.6	52.6	53.1
27	G	5 - 5	5 - 3	97	D	55.0	55.8	55.4
28	G	5 - 5	4 - 8	86	B	44.2	31.2	37.7
29	G	5 - 5	5 - 11	109	B	48.0	40.0	44.0
30	G	5 - 5	5 - 9	106	B	59.7	65.8	62.7
31	B	5 - 5	5 - 7	103	C	29.7	25.0	27.3
32	G	5 - 4	5 - 4	100	C	45.7	50.6	48.1
33	B	5 - 4	5 - 11	118	B	50.6	55.6	53.1
34	B	5 - 3	5 - 4	102	B	28.2	35.2	31.7
35	G	5 - 3	5 - 3	100	A	43.7	47.0	45.3
36	G	5 - 3	6 - 10	131	A	75.6	89.6	82.6
37	G	5 - 3	7 - 0	133	A	69.7	78.7	74.2
38	G	5 - 2	5 - 8	110	A	82.8	80.8	81.8
39	B	5 - 2	4 - 7	89	B	67.6	61.6	64.6
40	B	5 - 2	5 - 4	103	B	55.6	35.0	45.3
41	B	5 - 2	5 - 8	110	B	38.2	36.9	37.5
42	G	5 - 2	4 - 8	90	B	95.8	101.8	98.8
43	B	5 - 1	5 - 3	103	B	42.4	31.4	36.9
44	G	5 - 1	5 - 3	104	B	59.8	49.8	54.8
45	G	5 - 1	6 - 1	122	B	92.8	107.8	100.3
46	B	5 - 1	5 - 8	111	A	85.0	89.0	87.0
47	G	5 - 1	5 - 5	107	B	48.8	56.0	52.4
48	B	5 - 0	5 - 7	112	B	46.2	54.3	50.2
49	B	5 - 0	5 - 1	102	A	56.2	61.9	59.0
50	B	5 - 0	5 - 7	112	A	59.8	72.3	66.0

TABLE III

DATA FOR ANALYSIS IN NUMBER ABILITY OF FOUR-YEAR AGE-GROUP

Subjects	Sex	C.A.	M.A.	I.Q.	T.R. (number interest) (5)	Number-Test Score		
						First Trial (6)	Second Trial (7)	Average (8)
51	H	4 - 11	7 - 1	141	A	71.3	76.3	73.8
52	G	4 - 11	6 - 9	135	A	82.7	86.8	84.7
53	B	4 - 11	6 - 7	133	B	62.6	65.6	64.1
54	G	4 - 11	4 - 10	98	A	50.4	46.8	48.6
55	G	4 - 11	4 - 5	90	C	77.2	82.2	79.7
56	G	4 - 11	5 - 7	114	B	73.2	82.6	77.9
57	G	4 - 11	4 - 5	90	C	32.1	33.1	32.6
58	G	4 - 11	5 - 4	108	B	59.4	70.9	65.1
59	G	4 - 11	5 - 2	105	B	24.2	23.2	23.7
60	B	4 - 11	6 - 2	125	A	96.8	87.8	92.3
61	G	4 - 11	4 - 0	81	C	18.9	12.9	15.9
62	G	4 - 10	4 - 7	95	A	32.8	47.8	40.3
63	G	4 - 10	4 - 8	96	C	50.2	49.2	49.7
64	H	4 - 10	6 - 9	139	B	70.1	69.1	69.6
65	G	4 - 10	5 - 10	120	C	34.4	37.8	36.1
66	G	4 - 9	5 - 5	114	D	32.8	45.0	38.9
67	G	4 - 9	5 - 8	120	C	52.8	31.8	42.3
68	G	4 - 9	5 - 2	109	B	26.1	20.9	23.5
69	G	4 - 9	6 - 2	130	B	75.8	80.8	78.3
70	G	4 - 9	5 - 11	125	B	70.6	82.6	76.7
71	G	4 - 8	4 - 4	92	C	46.4	40.4	43.4
72	G	4 - 7	4 - 0	88	C	23.9	23.9	23.9
73	B	4 - 7	3 - 11	86	C	22.2	17.3	19.7
74	B	4 - 7	4 - 4	95	C	15.4	15.4	15.4
75	G	4 - 6	8 - 0	177	A	127.0	134.0	130.5
76	G	4 - 5	7 - 4	165	C	45.9	45.3	45.6
77	H	4 - 5	4 - 4	98	B	30.5	31.6	31.0
78	B	4 - 5	5 - 5	122	B	41.2	41.2	41.2
79	B	4 - 5	5 - 8	128	B	31.6	27.6	29.6
80	B	4 - 5	5 - 0	113	B	41.3	36.3	38.8
81	G	4 - 5	5 - 0	114	D	19.9	17.9	18.9
82	G	4 - 5	4 - 9	107	E	12.1	11.2	11.6
83	B	4 - 5	5 - 9	130	C	41.8	40.8	41.3
84	B	4 - 5	5 - 10	132	B	51.8	52.8	52.3
85	B	4 - 5	4 - 5	100	D	15.3	13.4	14.3
86	G	4 - 4	6 - 4	145	A	99.9	109.0	104.4
87	B	4 - 4	5 - 4	123	B	49.6	49.6	49.6
88	B	4 - 4	5 - 9	132	C	40.1	45.9	43.0
89	G	4 - 4	7 - 1	164	A	100.8	102.9	101.8
90	G	4 - 3	5 - 8	134	C	39.0	33.0	36.0
91	G	4 - 3	6 - 4	148	C	51.9	55.9	53.9
92	G	4 - 3	4 - 6	106	C	12.0	12.0	12.0
93	G	4 - 2	4 - 8	111	C	33.4	33.4	33.4
94	H	4 - 1	5 - 3	129	C	24.0	19.0	21.5
95	G	4 - 1	5 - 0	122	B	28.9	32.9	30.9
96	G	4 - 1	4 - 8	114	D	14.9	19.8	17.3
97	B	4 - 1	4 - 8	115	C	17.8	14.7	16.2
98	G	4 - 0	5 - 2	128	A	65.8	66.8	66.3
99	G	4 - 0	5 - 6	138	A	73.8	66.9	70.3
100	H	4 - 0	4 - 9	119	C	24.4	26.4	25.4

TABLE IV

DATA FOR ANALYSIS IN NUMBER ABILITY OF THREE-YEAR AGE-GROUP

Subjects	Sex	C.A.	M.A.	I.Q.	T.R. (number interest)	Number-Test Score		
						First Trial	Second Trial	Average
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
101	G	3 - 11	4 - 4	111	B	15.9	14.0	14.9
102	G	3 - 11	4 - 4	111	C	10.1	12.1	11.1
103	G	3 - 9	4 - 1	109	C	5.4	5.8	5.6
104	B	3 - 9	4 - 1	109	C	.5	4.3	2.4
105	G	3 - 9	5 - 2	138	B	28.7	29.7	29.2
106	G	3 - 8	4 - 8	125	B	14.5	15.7	15.1
107	G	3 - 8	4 - 3	111	C	4.7	7.6	6.1
108	B	3 - 7	4 - 10	135	C	18.8	17.8	18.3
109	B	3 - 7	4 - 2	116	C	8.8	7.4	8.1
110	B	3 - 6	3 - 10	109	E	6.5	8.5	7.5
111	B	3 - 6	5 - 0	143	A	18.2	14.9	16.5
112	B	3 - 6	4 - 6	128	C	12.9	12.6	12.7
113	B	3 - 5	4 - 7	134	B	21.0	17.5	19.2
114	G	3 - 5	4 - 5	129	B	12.1	11.2	11.6
115	B	3 - 5	3 - 6	102	C	9.5	5.5	7.5
116	B	3 - 5	3 - 10	112	C	20.8	17.8	19.3
117	B	3 - 5	3 - 6	102	C	18.8	16.9	17.8
118	B	3 - 3	4 - 8	143	C	9.7	7.9	8.8
119	B	3 - 3	3 - 9	115	C	20.0	15.9	17.9
120	G	3 - 2	4 - 2	131	B	33.6	35.7	34.6
121	B	3 - 2	4 - 3	134	C	14.8	14.7	14.7
122	B	3 - 2	3 - 7	113	C	4.4	8.4	6.4
123	B	3 - 2	4 - 2	131	C	15.7	15.9	15.8
124	B	3 - 1	2 - 11	94	D	5.7	5.3	5.5
125	B	3 - 0	3 - 8	127	E	8.4	6.5	7.4

employed in the course of the discussions presented in later chapters.

In filling out supplementary report blanks some teachers experienced considerable difficulty in rating children's number ability and explained their difficulty by the fact that specific instruction in number is not given in pre-first-grade years. When the teachers were reminded that they frequently call upon children to make responses in rhythms and to judge number and amount of pegs, splints, beads, blocks, and the like, some of them frankly declared that these activities are carried on without any conscious thought on their part of the number element involved. Others readily pointed out number values in daily school exercises but admitted inability to rate children in these. A few, however, rated the children promptly and the others, after discussing the problem with the examiner and other teachers and observing their pupils for a period of several weeks, rated all the children in

their respective groups.

All teachers used five points to differentiate ability: A represented superior; B, good or above average; C, average; and D, below average ability. E indicated retarded development.

Comparative Analysis of the Number-Test

The complete form of the number test is given in the Appendix, page 146, but in the discussion that follows the sixteen subtests are analyzed with reference, first, to content and, second, to similar tests used in other studies.

The test is arranged in three series: Series I contains three subtests in counting; Series II, five in recognition of small aggregates; and Series III, eight in combining aggregates. No duplicate form was designed, hence a retest was given after an interval of several days. Though certain of the subtests were too difficult for the youngest children, nevertheless, an opportunity to try each test was given to all children unless a succession of negative responses indicated the futility of such procedure. In instances of this kind a zero score was given for tests thus abandoned. Observations of performance as noted by the examiner were made on the test-forms and used in Chapter IV in the determination of group and individual development.

Series I: Subtests in Counting.- Series I contains three subtests, 1-3, and measures ability to repeat and apply the numeral series. Subtest 1 measures control of correct serial order of numerals and in the pages of this report this ability is designated rote counting; Subtest 2 measures ability to employ the terms of the numeral series, in a one-to-one relation, in discriminating items of an objective series, and, in the discussion that follows, this ability is called rational counting. The Binet-Simon scale includes a similar test in rational counting, namely, counting four pennies at five and twenty pennies at six years.

Observations were noted on the following types of reaction: (a) deviations from correct serial order; (b) synchronization of numeral terms and objective items; and (c) rhythmic counting tendencies interfering with the establishment of a one-to-one relation between items of the two series, e.g., 1, 2, 3 for the initial item discriminated and 4, 5, 6 for the next.

In the Stanford Revision of the Binet-Simon Scale, counting backward from twenty is the second commission in tests for eight-year-old children. Test 3 of the present series aims to determine to what extent this skill appears before the age of six.

Series II: Subtests for Recognition of Cardinal Number.- That children of school age have some acquaintance with group

number is patent to the most casual observer, but factors bringing about such acquaintance have only recently been subjected to analysis. Beckman,¹ Filbig,² Descoeudres,³ and, more recently, Buckingham and MacLatchy⁴ have attempted to distinguish certain factors in the development of cardinal number. Beckman distinguishes the following elements involved in recognition of cardinal number: (1) number building or reproduction, e.g., selecting the correct number in response to "Give me 5 ----"; (2) number distinction, responding correctly to "Is this or is that 5?"; (3) number finding, e.g., pointing out among multiple numbers all 4's or 6's; (4) number naming in response to the question, "How many pennies are here?"

Descoeudres, modifying exercises designed by Decroly and Degand,⁵ formulated a series of tests to determine "different kinds of grasp of cardinal number" and to ascertain the extent to which these differ in rate of development. Her tests made appeal to various senses: visual, auditory, and kinaesthetic.

Series II includes five subtests for group recognition--four, Subtests 4, 5, 6, and 8, based upon Beckman's analysis, and one, Subtest 7, suggested by Descoeudres' series.

Perception of likeness in size of aggregates is one of the constituent elements of group recognition. Subtest 4 was designed to measure its development by requiring children to match given sized groups of objects. Descoeudres⁶ used a similar form for numbers 1-4, but permitted the child to observe the examiner as she formed the group. Baldwin and Stecher⁷ applied Descoeudres'

¹Hermann Beckmann, "Die Entwicklung der Zahlleistung bis 2-6 jährigen Kindern," Zeitschrift für Angewandte Psychologie, XXII (March, 1923), 2-72.

²Josef von Filbig, "Untersuchungen über die Entwicklung der Zahlvorstellungen im Kinde," Zeitschrift für Pädagogische psychologie und Experimentelle Pädagogik, XXIV (1923), 105-13, 156-68.

³Descoeudres, Alice, Le développement de l'enfant, pp. 7-14, 233-80. Paris: Delachaux & Niestle, S. A. (no date).

⁴B. R. Buckingham and J. MacLatchy, op. cit., pp. 473-524.

⁵O. Decroly and Julia Degand, "Observations relative à l'évolution des notions de quantités, continues et discontinues chez l'enfant," Archives de Psychologie, XII (May, 1912), 81-121.

⁶Descoeudres, op. cit., Test X.

⁷Bird T. Baldwin and Lorle L. Stecher, The Psychology of the Pre-School Child, p. 155. New York: D. Appleton & Co., 1925.

technique for numbers 1-10 with one modification; they required the child to "hide his eyes." This latter plan, tried out in the period of preliminary experimentation, was found unsatisfactory because some children pressed their hands so firmly over their eyes as to cause serious distraction.

Subtest 5 aims to measure another factor in group recognition, namely, ability to discriminate aggregates differing slightly in size. Beckmann's test for this factor he calls "number-distinction" and Stern describes it as follows: "A group of 2 and a group of 3 are successively pointed out with, 'Is this 2 or is that?' Or there is only a group of 3 on the table, 'Is that 2 or 3?'"¹ With the children of the present investigation number names were not employed at this point. In order to determine whether they could discriminate one given number from another, when not distracted by numeral terms, they were required to sort a pack of cards containing duplicate cards for two numbers. Cards, presenting numbers by a grouping of dots, were exposed in random order.

Subtest 6 is one in which children form number groups in response to such requests as, "Give me five sticks of candy." In making responses they must distinguish five as greater than numbers less than 5 and as less than six and numbers greater than six. In this form the test is identical with that used by Beckmann² who calls it "number-building" and classifies it as the easiest of the four used by him. Buckingham and MacLatchy³ following the same procedure reached like determination as to comparative difficulty but report that the degree of difficulty between this and the most difficult phase, number naming, is but slight and much less than indicated by Beckmann.

The same principle in Descoeudres' series (Test VII) takes a somewhat different form. With a pile of cubes, stones, or the like before her, the examiner says, "See these stones. Well, you are to take one for me and one for you." Making note of how the subject proceeds, the examiner continues, "Now, you take three of them for yourself and three for me." She makes similar requests involving two, five, and four.⁴ Baldwin and Stecher,⁵ using Beckmann's technique, found that children who knew the meaning of

¹William Stern, op. cit., p. 415.

²Hermann Beckmann, op. cit., pp. 1-72.

³B. R. Buckingham and J. MacLatchy, op. cit., p. 487.

⁴Alice Descoeudres, op. cit., p. 253, Test IV.

⁵Bird T. Baldwin and Lorle L. Stecher, op. cit., p. 161,

number names reacted quickly, even checking their performance by counting, but those not knowing higher numbers seemed to be conscious of the fact, for "they grabbed a handful of counters" with an air that indicated it was hopeless to count.

Subtest 7 measured ability to reproduce the number of sounds heard. The foregoing tests presented number groups in simultaneous exposure and made visual or motor appeal; Subtest 7, on the contrary, presented group number by a series of successive auditory stimuli. Responses at all age-levels indicate that this test is much more difficult than the other tests for group recognition. Two investigators,¹ using practically the same form of the test, attribute its difficulty to a lack of visual clues and a requirement of greater concentration of attention. Descoeudres,² on the theory that this aspect of group recognition is dependent upon a sense of rhythm, used a rhythmic hand clapping instead of bells. She clapped her hands while the child's back was turned and required him to respond with a like number of claps. She found that fatigue entered quickly and hence made it difficult to judge between the part played by this factor and that by inability. She concludes, however, that auditory ideas of number are slower in development than visual number.

The task in Subtest 8 is one requiring quick perception and prompt naming of aggregates presented on cards in timed exposure. The aggregate may be perceived as a whole, or as a smaller aggregate to which other units within the whole are added one by one, or it may be perceived as a series of units and counted from memory. In form, the test resembled that used by Brownell³ though grouping was such as to favor regularity of pattern, e.g., the domino quincunx was used for 5, the triangle for 3, the quadratic pattern for 4, and so on. The test is like that of Beckmann but differs from it by employing a time exposure technique similar to that of Filbig.⁴ Buckingham and MacLachy⁵ used Beckmann's technique but avoided "pattern arrangement" because of its effect in aiding number perception.

¹Ibid.

²Descoeudres, op. cit., pp. 256-59.

³William A. Brownell, The Development of Children's Number Ideas in Primary Grades, p. 9 ff. Supplementary Educational Monographs, No. 35. Chicago: Department of Education, University of Chicago, 1928.

⁴Filbig, op. cit., pp. 105-13, 156-68.

⁵Buckingham and MacLachy, op. cit., p. 487.

Series III: Subtests in Combining Small Aggregates.- Series III contains eight subtests in combining aggregates under four type conditions: Subtests 9-12 contain ten items each in combining two numbers into sums less than ten; Subtests 13-16, five items each in combining three numbers likewise into sums less than ten. The combinations in Subtests 9, 10, 13, and 14 were presented by means of small light colored discs in a manner that avoided pattern arrangement. Correct and incorrect responses and peculiarities of method were recorded. The combinations used in Subtests 9-12 are as follows:

2 and 1	6 and 3	5 and 3	5 and 4
5 and 2	8 and 1	6 and 2	4 and 4
7 and 2		4 and 3	

In Subtest 9, designated "Visible Number" in this report, the aggregates forming the combinations are successively presented and left exposed during the entire time of subject's attempt at combining. In Subtest 10, called "Hidden Number," number groups are exposed for two seconds only. Many children who deal readily enough with the tasks in Subtest 9 fail in 10, for the presence of the visible objects facilitates combining by counting. Similarly in Subtest 13, combining three visible aggregates, the process is performed by children who are unable to combine the same numbers under the condition of limited time exposure.

Test 11 uses the same ten combinations but employs no concrete counters. A child is asked to hold out both hands with palms up. The examiner goes through the motions of putting a group of pennies in each hand. For example, she says, "Here are two pennies for this hand and here is one for this one. Close your hand so they won't roll away and tell me how many you have." In carrying out the test it is necessary that the child give strict attention and that he visualize the situation as the counters are imaginatively but impressively placed in his hand. This test is referred to in data that follow as "Imagined Number."

Test 12, designated "Abstract Number" in this report, uses no concrete number. The examiner says, "We shall not use any pennies this time at all. You tell me right off your answer. Two and one more? Five and two more? Four and four more?" and so on with the remaining combinations. Characteristics of methods and all responses are recorded.

The second part of the series includes Subtests 13-16 and measures ability to combine three aggregates under the same four-type conditions used with two aggregates. Adding an aggregate to the sum of the first two, i.e., holding an unseen number in mind and adding on another is one of the outstanding difficulties in

column addition with young children.¹ The combinations used for this purpose are as follows:

5 and 1 and 2

4 and 3 and 2

4 and 2 and 1

4 and 2 and 2

1 and 2 and 3

The three series of tests were designed to determine to what extent children of these early ages can recognize and use number ideas in the several processes described in the foregoing pages. Series I aims to answer questions regarding mastery, or lack of it, in serial number. Series II seeks an analysis of elements of growth in control of aggregate number. Series III tests the beginnings of ability to combine small groups of aggregates.

Method of Scoring the Number-Test

The number of responses within the subtests varied; Subtests 4-8 included seven items each; 9-12, ten items each, and 13-16, five each. The thirteen tests, therefore, yielded approximately ninety-five responses. Any method of scoring the results of such tests must be more or less arbitrary; the method that seemed fairest from the standpoint of the purposes of the investigation was that which gave a score of one for each correct response.

Subtests 1, 2, and 3 presented a different problem for scoring. These tests dealt with counting: rote, rational, and backward. In Subtests 1 and 2 the number in the series correctly counted, without intervening error, was divided by ten and the quotient thus obtained was used as the score. Weighting to this extent seemed justified because the counting process was much less difficult than the processes of the other subtests. Subtest 3, counting backward, because of greater difficulty involved, was scored 5, 10, or 20, according to the point from which successful counting was done. That this method was too generous is shown later in the analysis of separate tests (see Chapter IV, Figure 10).

The total score for each child, obtained from scores for separate subtests, appears in Tables II-IV, columns 6 and 7. Table II presents scores for five-year children arranged in descending order of chronological age; similarly, Table III presents those for four-year, and Table IV, those for three-year olds. Data for four- and five-year groups are used in the statistical evaluation of the

¹Edward L. Thorndike, The Psychology of Arithmetic, p. 52. New York: The Macmillan Co., 1924.

number-test which immediately follows in this chapter. Data for all three successive age-groups are used in the next two chapters to determine (1) the relation of general intelligence to number ability, and (2) to analyze age-progress in counting, group recognition, and combining groups.

Statistical Evaluation of the Test

Evaluation for Reliability.- The test was evaluated for reliability by means of correlation methods. Correlation between the two sets of scores, obtained from two administrations of the test and given in columns 6 and 7 of Tables II and III, yielded a reliability coefficient of $.96 \pm .01$. Range of ability within the two years included in the scores had but slight effect on the magnitude of the coefficient of correlation. The coefficient of reliability computed for each age separately was the same for both year groups, $.95 \pm .01$.

Correlation data are shown in Table V and in the scatter diagram of Figure 1. Scores for the first testing, Score A, are read from intervals at the left and those for the second testing, Score B, from intervals at the top of the table. Data in Figure 1 are read in a corresponding manner.

Ruch and Stoddard interpret a coefficient of .95 as very high and indicative of a large proportion of common elements or causes in the two performances.¹ The coefficients for year groups considered separately and together are higher than those demanded by most intelligence tests now in current use,² and higher by a few points than those ordinarily obtained from correlations between educational tests.³

Two possible explanations offer. One concerns itself with the fact that the reliability coefficient was derived from two sets of scores obtained from a single form of the test. Kelley⁴

¹G. M. Ruch and G. D. Stoddard, Tests and Measurements in High School Instruction, pp. 55-6. New York: World Book Co., 1927.

²Henry E. Garret, Statistics in Psychology and Education, pp. 268-69. New York: Longmans, Green & Co., 1926.

³Walter S. Monroe, An Introduction to the Theory of Educational Measurements, p. 244. Boston: Houghton Mifflin Co., 1923.

⁴Trumen L. Kelley, Interpretation of Educational Measurements, pp. 39-40. New York: World Book Co., 1927.

TABLE V

CORRELATION BETWEEN TOTAL SCORE A AND TOTAL SCORE B
OF NUMBER-TEST (RELIABILITY)

Score B	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99	100-109	110-119	120-129	130-139	Total
Score A														
130-139													2	2
120-129											1	1		2
110-119										4				4
100-109								1	1	3				5
90- 99								3	1					4
80- 89						2	1	5			1			9
70- 79						7	2							9
60- 69			2	5	6	2	2							17
50- 59			3	9	4									16
40- 49		2	7	4										13
30- 39	2	7	2											11
20- 29	8													8
10- 19														
Total	10	9	14	18	10	11	5	9	2	7	2	1	2	100

Number of Cases 100; S.D. (Total Score A) = 2.69; S.D. (Total Score B) = 2.96; r_{12} = .96; P.E. = $\pm$.012.

points out that coefficients thus derived are, in general, numerically higher. Foran¹ reports that magnitude of correlation coefficient is one and five-tenths points greater when the same form is repeated than when duplicate forms are employed.

Data in Table V, Figure 1, indicate that group scores for the first and second trials did not vary greatly. This relationship would seem to support the "practice" explanation given in the foregoing statements from Kelley and Foran. A study of the observations entered on individual test-form records, however, shows that correct responses on both trials were given promptly and with sureness and that negative responses were given in a similar

¹T. G. Foran, "A Note on Methods of Measuring Reliability," Journal of Educational Psychology, XXI (May, 1931), 383-87. (Morison-McCall Spelling Scale in Grades II-VIII was used in four separate schools.)

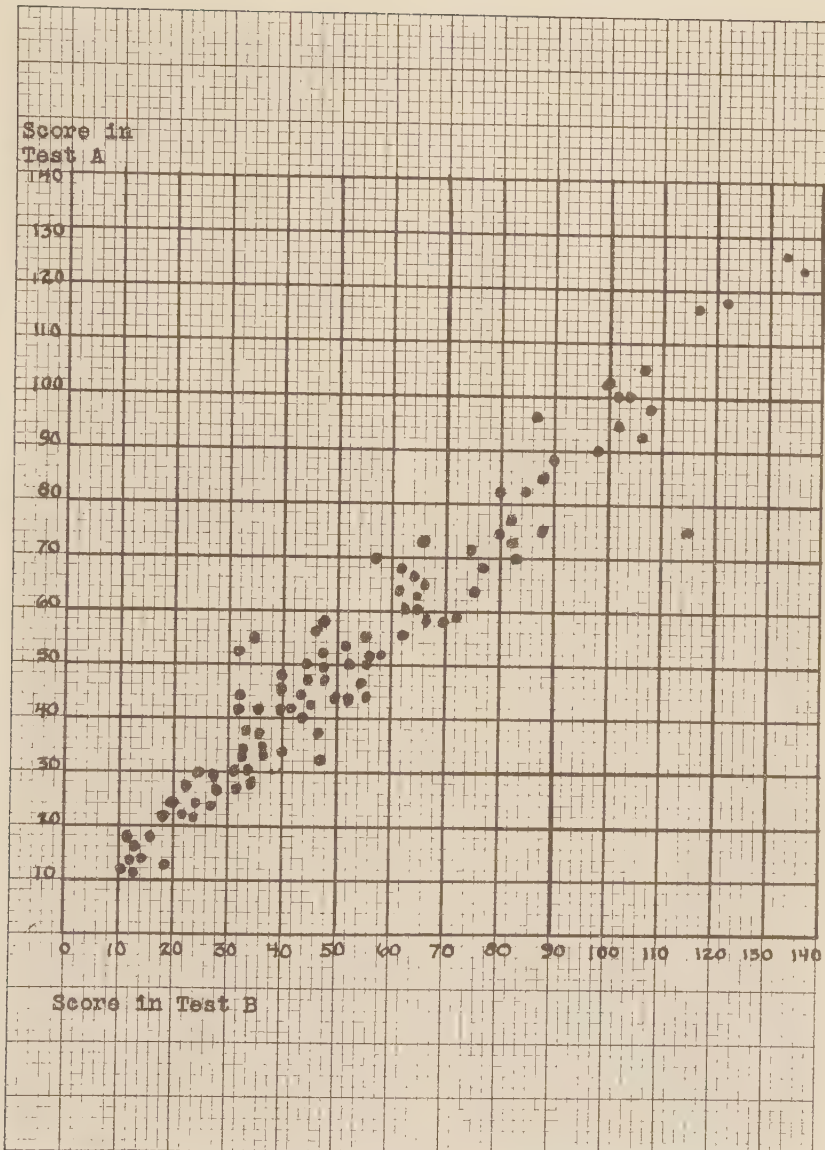


Fig. 1.- Reliability of Number Test as Indicated by Two Testings of 100 children — First test, A; Second test, B (Total Scores indicated).

manner. The character of the responses clearly indicated they were prompted by what the subjects knew or felt they did not know.

The possibility of practice effect and the feasibility of obviating it were considered in early stages of the investigation, but the nature of the test items made a duplicate form practically impossible. For example, there are no corresponding substitutes to use with immature children for such items as rote counting by ones, or the combining of simple aggregates as 3 and 2.

A second explanation of the magnitude of the reliability coefficient may be fallibility in method of recording, scoring, and computing measures. The method used was so simple and uncomplicated that it was not thought necessary to have a second or third scorer recheck the results. Nevertheless, errors at this stage may explain, in part, the magnitude of the reliability coefficient. In summary it may be said that, though the magnitude of the reliability coefficient as noted, is in part influenced by range of talent and possibly by the use of retest instead of duplicate-test scores, nevertheless observational data of test performance indicate a high degree of stability in the functions measured.

Additional evidence of reliability is offered by the index of reliability and by the values for the standard and probable errors of measurement. Data for these indices computed by the several formulas are as follows:

$$\begin{aligned} r(\text{obt. true}) &= .98 && (\text{Index of reliability})^1 \\ \text{S.D.}(\text{est. obt.}) &= 2.41 && (\text{Standard error of measurement})^2 \\ \text{P.E.}(\text{est. obt.}) &= 1.62 && (\text{Probable error of measurement})^3 \end{aligned}$$

Since 1.0 represents perfect correspondence, the index of reliability is very high. It is interpreted as the maximum value that the reliability coefficient can have in correlation between the scores of the test series and subjects' true scores, that is, average scores derived from multiple measurements by the same or duplicate tests. Kelley says,

"The highest possible correlation which can be obtained (except as chance might occasionally lead to higher spurious correlation) between a test and a second measure is with that which truly represents what the test actually measures,—that

$$^1 r(\text{obt. true}) = \sqrt{r_{12}}$$

$$^2 (M) = 1 \sqrt{1 - r_{12}}$$

$$^3 \text{P.E.}(M) = .6745 \sqrt{1 - r_{12}}$$

is, the correlation between the test and the true scores of individuals in just such tests."¹

The standard and probable errors of measurement indicate the divergence obtained from probable true scores. From the standard error it is obvious that the chances are 68 in 100 that true scores fall within the limits of obtained scores ± 2.41 points; or practical certainty places them within the limits of obtained scores $\pm 3 \times 2.41$. The probable error of measurement indicates that the chances are even that subjects' obtained scores deviate from their corresponding true score by not more than ± 1.62 points.

Correlations of Test Scores with Teachers' Ratings.— Score A was correlated with teachers' ratings of number ability. The data appear in Table VI. The coefficient of correlation found was

TABLE VI

CORRELATION BETWEEN TOTAL SCORE A AND TEACHERS' RATINGS

Teachers' Ratings	E (50-59)	D (60-69)	C (70-79)	B (80-89)	A (90-99)	Total
Score A						
120-129					2	2
110-119				1	1	2
100-109					4	4
90- 99				2	3	5
80- 89					4	4
70- 79			1	4	4	9
60- 69			1	6	2	9
50- 59		①	⑤	⑦	④	17
40- 49			6	9	1	16
30- 39		①	5	5	②	13
20- 29			7	4		11
10- 19	1	3	4			8
Total	1	5	29	38	27	100

Number cases 100; S.D. (Total Score A) = 2.69; S.D. (Teachers' Ratings) = .91; $r = .65$; P.E. = $\pm .038$.

$.65 \pm .038$. This coefficient indicates a fair degree of correspondence between ability as estimated by the teachers and as

¹T. L. Kelley, "The Reliability of Test Scores," Journal of Educational Research, III (May, 1921), 370-80.

measured by the tests. The agreement is surprising in view of rather wide discrepancies in certain individual subjects' ratings, e.g., of three subjects with number-test scores in the 30-39 interval and indicated in Table VI by encircled figures, two are ranked 'A' and one 'D' by their respective teachers. Of ten others having the same score, five are ranked 'B' and five 'C.' Divergences of a similar nature occur in interval 50-59; four subjects are rated 'A,' seven 'B,' five 'C,' and one 'D,' nevertheless all attained approximately the same score on the number-test. On the other hand, no subject making a high score was rated low by the teachers. In spite of the foregoing discrepancies number-test scores show a fair agreement with teachers' judgment of number ability and give a basis for the inference that the test, in general, has a predictive value considerably more reliable than chance.

Correlation with Other Criteria.- In order to determine to what extent number ability is associated with chronological age and with general intelligence, correlations were computed with the following data: (a) chronological age; (b) I.Q.; and (c) M.A. (Detroit Kindergarten and Stanford-Binet considered together and separately). Correlations between number-test scores and chronological ages were computed for the combined four- and five-year age-group and for each year separately. Sixty of the one hundred subjects had been given the Stanford-Binet test and forty, the Detroit Kindergarten test. Correlations of number-test scores and mental ages for these two groups were obtained for the two M.A. groups separately and for the two groups combined. The amount of agreement or correspondence is shown in Table VII.

Correlation is lowest with chronological age. It is $.35 \pm .05$ when the two groups are considered together and even less when considered separately, $.26 \pm .08$ for the younger and $.24 \pm .08$ for the older. Correlation is less with chronological age though the difference is not great. The values for r as given in Table VII indicate that other factors functioning in the ability measured outweigh the factor of age.

There is a marked higher value of r in all correlations involving M.A. or I.Q. The highest is $.71 \pm .04$ when number ability is correlated with mental age scores of the Stanford-Binet taken separately from those of the Detroit Kindergarten. Correlation with the latter is $.48 \pm .08$ but when the two are combined a higher coefficient is obtained, that is, $r_{12} = .63 \pm .04$. These values for r_{12} indicate the presence of a fairly large proportion of common elements in the functions measured by the intelligence tests and the number-test. The correlation of the latter with I.Q. is somewhat higher than that with chronological age, but much less

TABLE VII

CORRELATION BETWEEN TOTAL SCORE A AND OTHER
CRITERIA: C.A., M.A., AND I.Q.

Criteria	Combined Age-Groups (48-72 mo.)			Separate Age-Groups (48-59; 60-72 mo.)			
	No. Cases	r_{12}	P.E.	No. Cases	r_{12}	P.E.	Age-Group
N.T. (number-test score) and C.A.	100	.35	.05	50	.26	.08	48-59 mo.
				50	.24	.08	60-72 mo.
N.T. and I.Q.	100	.40	.05				
N.T. and M.A. (Detroit Kindergarten)	40	.48	.08				
N.T. and M.A. (Stanford Binet)	60	.71	.04				
N.T. and M.A. (Stanford Binet and Detroit Kindergarten)	100	.63	.04				

than with M.A. Since mental age enters into the constitution of I.Q., there are doubtless other factors influencing the relation. A discussion of this problem will be deferred until Chapter IV.

Comparison of Median Age-Group Scores with C.A., M.A., and I.Q.- A study was made of the relation between median scores and chronological age. The range in age for the two older groups was divided into eight equal intervals of three months each and median number-test scores for each interval were computed. The data (Table VIII) indicate a general increase in median score between the two extreme age intervals; the lowest is 26.7 and the highest 65.0 points. Intervals in between show several marked deviations from general trend, e.g., in intervals 4;6-4;8¹ and 5;3-5;5 median scores are considerably less than in intervals just preceding.

Two similar studies were made of the relation between number test scores and mental ages and intelligence quotients. The data for M.A. are presented in Table IX and that for I.Q. in Table X. The position of the median total number-test score for each

¹Form for expressing age in years and months.

TABLE VIII

COMPARISON OF MEDIAN SCORE WITH C.A.

C.A.	4;0- 4;2	4;3- 4;5	4;6- 4;8	4;9- 4;11	5;0- 5;2	5;3- 5;5	5;6- 5;8	5;9- 5;11	Total
Total Score A									
120-129			1					1	2
110-119							1	1	2
100-109		1					3		4
90- 99		1		1	2		1		5
80- 89				1	2		1		4
70- 79	1			6		1	1		9
60- 69	1			1	1	1	(2)	(3)	9
50- 59		(2)		(4)	(4)	(4)	1	2	17
40- 49		6	1		3	4	2		16
30- 39	1	3		4	1		4		13
20- 29	(3)		(2)	2		2	2		11
10- 19	2	4	1	1					8
Total	8	17	5	20	13	12	18	7	100 N
Median	26.7	52.5	27.5	57.5	56.3	50.0	60.0	65.0	

TABLE IX

COMPARISON OF MEDIAN SCORE WITH M.A.

M.A.	3;6- 3;11	4;0- 4;5	4;6- 4;11	5;0- 5;5	5;6- 5;11	6;0- 6;5	6;6- 6;11	7;0- 7;5	7;6- 7;11	8;0- 8;5	Total
Total Score A											
120-129 .								1		(1)	2
110-119 .					1					1	2
100-109 .							3	1			4
90- 99 .			1			4					5
80- 89 .					2	○	1	1			4
70- 79 .		1			3	2	(2)	1			9
60- 69 .			2	2	1		2	(2)			9
50- 59 .			2	6	(6)	1	2				17

TABLE IX (Continued)

M.A.	3;6- 3;11	4;0- 4;5	4;6- 4;11	5;0- 5;5	5;6- 5;11	6;0- 6;5	6;6- 6;11	7;0- 7;5	7;6- 7;11	8;0- 8;5	Total
40- 49 .		1	1	(7)	4	1		2			16
30- 39 .		(2)	(3)	3	4			1			13
20- 29 .	(1)	1	1	6	2						11
10- 19 .		3	4	1							8
Total . .	1	8	14	25	23	8	10	9		2	100 N
Median .	25.0	30.0	37.3	43.6	50.9	55.8	62.5	67.5		120.0	

TABLE X

COMPARISON OF MEDIAN SCORE WITH INTELLIGENCE QUOTIENT

I.Q.	80- 89	90- 99	100- 109	110- 119	120- 129	130- 139	140- 149	150- 159	160- 169	170- 179	Total
Total Score											
120-129 .					1					(1)	2
110-119 .			1			1					2
100-109 .				2	1				(1)		4
90- 99 .		1	1		2		1				5
80- 89 .				2		2					4
70- 79 .		1		2	1	4	(1)				9
60- 69 .	2	1	1	2	1	(2)					9
50- 59 .		(5)	5	(3)	(2)	1	1				17
40- 49 .	1	1	(5)	3	2	3			1		16
30- 39 .	(2)	4		3	2	2					13
20- 29 .	3		5	1	2						11
10- 19 .	1	1	3	3							8
Total . .	9	14	21	21	14	15	3	0	2	1	100 N
Median .	37.5	52.0	45.0	56.7	55.0	67.5	75.0		90.0	120	

group is indicated by circles and scores at the bottom of the respective columns show general increase in value.

The foregoing results present graphically the influence of several factors. The median performance improves with a considerable degree of regularity from age to age. The median score, when C.A. is the criterion, increases from 26.7 at 4;1, to 65.0 at 5;10. Correspondingly, with M.A. as criterion, the rise in attainment as represented by increase in median score is from 25.0 at M.A. level of 3;9, to 120 at M.A. level of 8;3. With I.Q. as comparison criterion, the median total score rises from 37.5 at I.Q. 85, to 120.0 at I.Q. 175. Though the number of scores for I.Q.'s above 140 is small, nevertheless the general trend indicates that ability as measured by the number test is influenced more by mental age and general intelligence than by chronological age.

Significance of Correlations and Comparisons

The statistical analysis of obtained measures by methods of correlation and comparison has yielded data for the following deductions:

(1) The reliability of the number test is very high. The correlation between the two sets of scores, Score A and Score B, gives the coefficient of reliability as, $r_{12} = .96 \pm .012$. The high correlation between the two sets of scores indicates a high degree of constancy in functions measured. The extent to which high correlation evidences apparent stability is obvious, but whether or not this is the same as the measure of actual stability is not shown by the data at hand. It may be argued on good authority¹ that high correlation is the best evidence available at the present time for establishing reliability.

(2) Correlation between teachers' ratings and total Score A gives a coefficient of correlation of .65 with a P.E. of $\pm .038$.

(3) A study, by means of correlation methods, of factors influencing number ability showed that the latter is more closely correlated with mental age and general intelligence than with chronological age. Furthermore, a comparison of median scores in the number-test with each of the three criteria supports this finding.

(4) The foregoing conclusions give rise to the problem presented in the next chapter, namely, determination of the extent and potency of general intelligence in relation to ability in dealing with number experiences.

¹Karl J. Holzinger, Statistical Methods for Students in Education, p. 168. Boston: Ginn & Co., 1928.

CHAPTER III

GROWTH OF NUMBER ABILITY IN RELATION TO AGE- PROGRESS AND TO GENERAL INTELLIGENCE

The Problem.- The preceding chapter showed that number ability is more closely related to general intelligence than to chronological age. The problem of the present chapter is twofold: it seeks to analyze growth in ability to deal with number experiences at successive age levels from three to six years in subject groups (1) of comparable brightness, and (2) of variable brightness. The purpose, in other words, is to determine age-progress in number ability and the relation of such ability to general intelligence.

Equating Brightness Factor in Age-Groups.- Table XI and Figure 2 present the distribution of I.Q. scores for all subjects in Classes Two and Three. An examination of these data shows that the three groups as constituted do not represent a balanced distribution of general intelligence. The disproportionate number of scores in some intervals, and the presence in some years, and absence in others, of excessively high or unusually low scores yield age-groups unequally weighted in brightness. The fourth year, for example, has five scores higher than the highest scores in third and fifth years. The latter, furthermore, has greater concentration of scores in lower intervals than appears in either of the other two years.

In order to equate groups for purposes of comparison, the intelligence factor was made comparable. Comparability was secured by eliminating scores in the higher intervals, 145-184, and reducing the number of disproportionate scores in other intervals. It should be noted that this procedure reduced the number of subjects from one hundred twenty-five to eighty-six as follows:

5;0-5;11	40 subjects
4;0-4;11	33 subjects
3;0-3;11	13 subjects

Data for comparable age groups, determined in the manner thus described, appear in Table XI in the second column for each age group. The population of the oldest group is approximately three times, and that of the next oldest two and one-half times that of the youngest group. The population in the youngest group

TABLE XI

I.Q. DISTRIBUTION IN ORIGINAL AND IN COMPARABLE
3-6 YEAR AGE-GROUPS

Intelligence Quotients	Age - Groups						Combined	
	3.0 - 3.11		4.0 - 4.11		5.0 - 5.11		Age-Groups	
	Original Group	Comparable Group	Original Group	Comparable Group	Original Group	Comparable Group	Original Group	Comparable Group
175-184			1				1	
165-174			1				1	
155-164			1				1	
145-154			2				2	
135-144	4	0	4	1	2	1	10	2
125-134	7	1	11	4	4	4	22	9
115-124	3	2	7	6	9	9	19	17
105-114	8	7	11	11	14	12	33	30
95-104	2	2	6	6	11	9	19	17
85- 94	1	1	5	4	9	4	15	9
75- 84	0	0	1	1	1	1	12	2
Total	25	13	50	33	50	40	125	86
Median		110.0		110.0		110.0		

is too meager to warrant refinement in method of analysis possible in the other groups, but the number, small as it is, will serve to indicate the presence or absence of consistent trends. The three distributions, represented in Figure 3, page 34, have a common median I.Q. of 110.

In summary it should be noted that the comparable age-groups, though unequal in population, have a bilateral symmetrical distribution of brightness with a median I.Q. slightly above average general intelligence. Data for the restricted population of the youngest group, inadequate for statistical validity in a rigorous sense,¹ are functionally significant for the problem under consideration. Subject to the limitations set forth the groups are

¹Karl J. Holzinger, op. cit., p. 232.

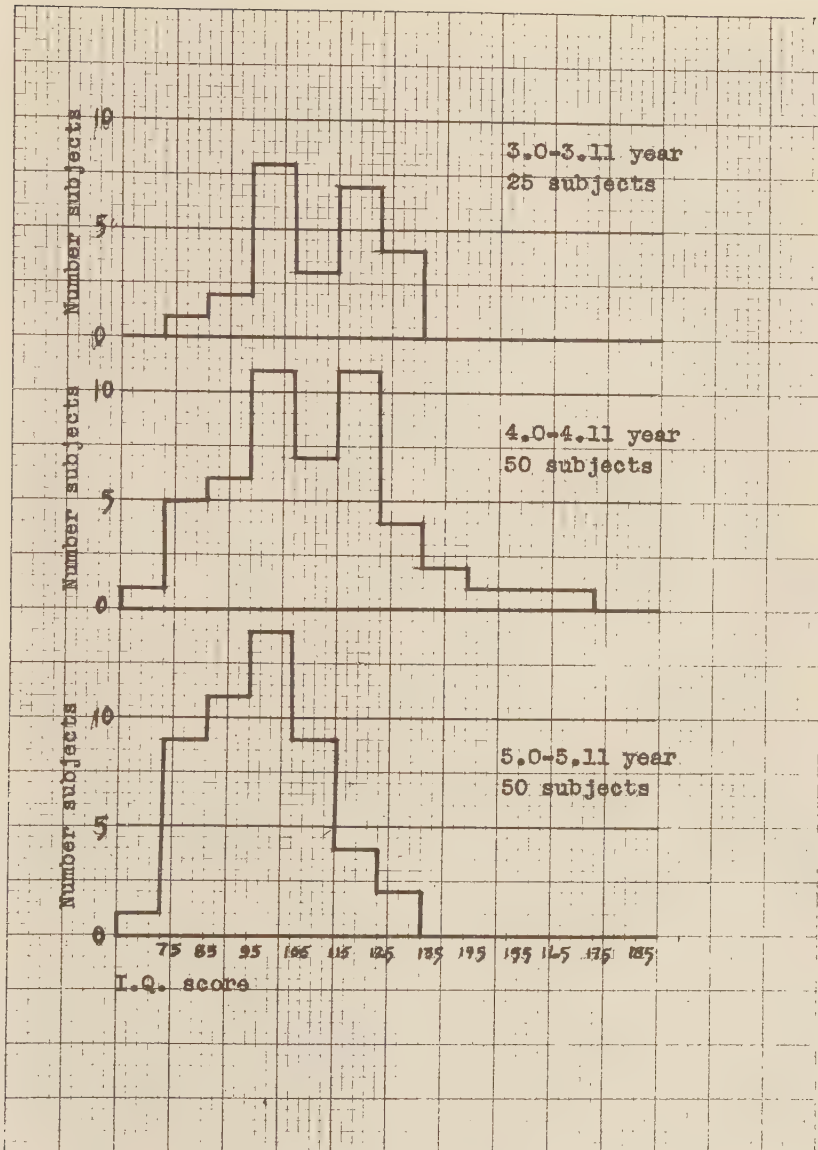


Fig. 2.- Comparison of Intelligence Quotients of 3-6 Year Age-Groups (125 subjects).

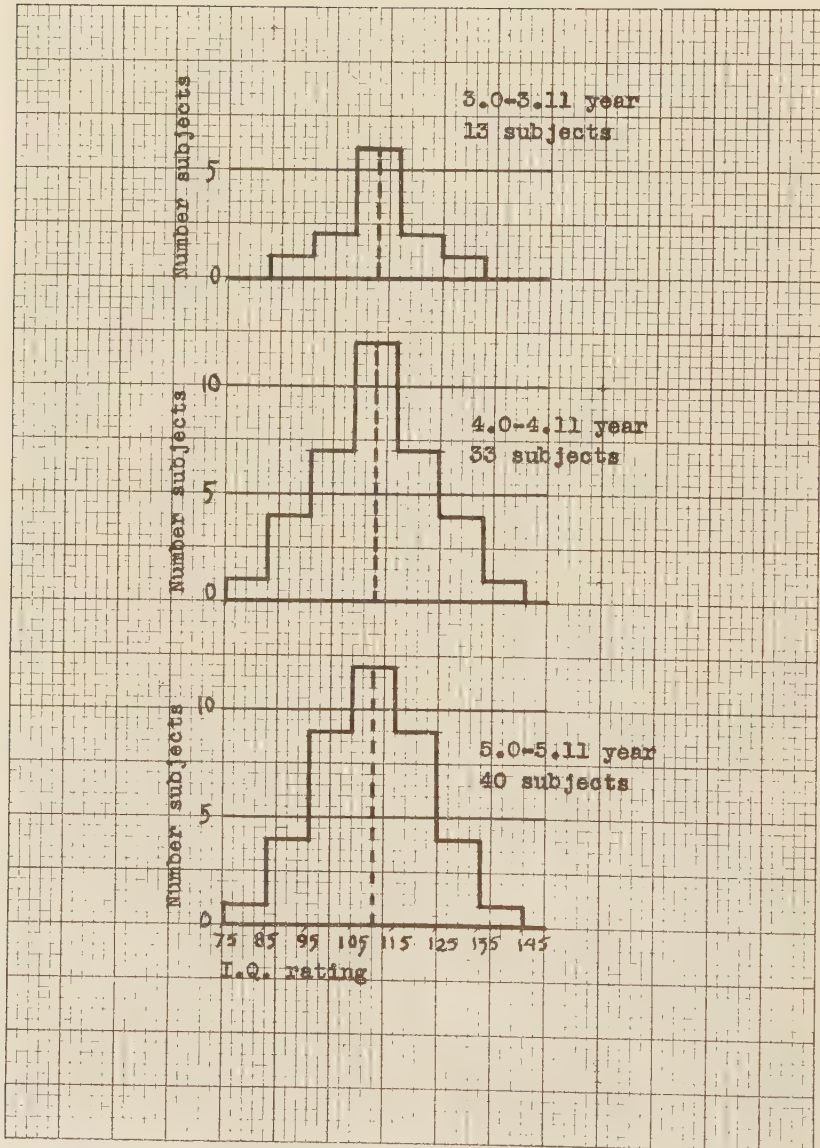


Fig. 3.- Comparison of Intelligence Quotients of 3-6 Year comparable Age-Groups (86 subjects).

designated as homologous or comparable in the discussion that follows and interpretations are made with these limitations in mind.

Reliability of Obtained Measures

Individual and group scores obtained from administration of the number-test give the basic data for analysis of number ability. These scores for comparable age-groups appear in Tables XVIII-XX in the next chapter, pages 50-53. They have been so placed because their inclusion in the text at this point would impede the exposition; essential data contained therein have been compiled in Table XII which immediately follows. At the bottom of the table are the following obtained measures: number of cases, means, medians, and standard deviations.

(a) Reliability of Means.- How reliable are the obtained means? How closely do they approximate the same functions in the three to six-year population at large? The sampling error of measurement gives a measure of reliability, or probable deviation, between obtained and true means. In accord with the limitations previously pointed out, sampling reliability coefficients are interpreted as representing deviations between obtained and true measures of like superior three-six year old children in the population at large. Interpretation is further limited in the case of the youngest group by the factor of population, as aforementioned. Nevertheless, it is reasonable to assume that its inadequacy is minimized by the fact that distribution of general intelligence in the youngest group is comparable to that in the two older groups.

Reliability formulas were applied to measures for groups above and below the median I.Q. While the population in these groups was too small for reliable determinations, it is assumed, however, that any degree of reliability established for measures of entire age-groups automatically establishes corresponding reliability for the partial age-groups above and below the median of general intelligence.

The standard errors of the means for entire age-groups, given in Table XIII, indicate that obtained means probably do not diverge from true means more than plus or minus a few points. Data showing limits within which probable true means lie appear in the last column of the table.

A more exacting criterion of sampling reliability places obtained means within three standard deviations of true means. Table XIV gives upper and lower limits within which, according to this more rigorous demand, the true means lie.

TABLE XII

DISTRIBUTION OF TOTAL SCORES BY AGE AND I.Q. GROUPS

Total Score	Ages 3-0 to 3-11			Ages 4-0 to 4-11			Ages 5-0 to 5-11		
	Low	High	All	Low	High	All	Low	High	All
	I.Q. Group	I.Q. Group		I.Q. Group	I.Q. Group		I.Q. Group	I.Q. Group	
120.0-124.9								1	1
115.0-119.9								1	1
110.0-114.9							1		1
105.0-109.9									
100.0-104.9								1	1
95.0- 99.9								2	2
90.0- 94.9					1	1	1		1
85.0- 89.9							1	2	3
80.0- 84.9								1	1
75.0- 79.9									
70.0- 74.9								(2)	2
65.0- 69.9					2	2		1	1
60.0- 64.9					2	2	3	1	4
55.0- 59.9				1		1	2	3	5
50.0- 54.9							(3)	2	5
45.0- 49.9				3	3	6	1	2	3
40.0- 44.9					○		3	1	4
35.0- 39.9					2	2	1		1
30.0- 34.9				2	2	4	1		1
25.0- 29.9				(2)	2	4	2		2
20.0- 24.9		2	2	3	2	5	1		1
15.0- 19.9	1	(1)	2	3		3			
10.0- 14.9		1	2	2	1	3			
5.0- 9.9	(3)	1	4						
0.0- 4.9	2	1	3						
Total . . .	6	6	13	16	17	33	20	20	40
Md.	6.66	15.0	9.9	25.0	40.0	32.5	51.6	67.5	58.0
M.	7.5	14.16	10.96	29.06	41.87	36.75	53.75	74.25	64.0
S.D. . . .	5.0	6.85	6.90	13.55	21.35	18.85	22.2	23.37	24.9

TABLE XIII

PROBABLE TRUE MEANS OF LIKE THREE-SIX-YEAR
AGE GROUPS IN POPULATION

Age Groups	No. Cases	Obtained Means	Obt. S.D.	Standard Error of Mean	Probable True Mean
3;0-3;11	13	10.96	6. 9	± 1.91	12.81 9.09
4;0-4;11	33	36.75	18.85	± 3.28	40.03 33.47
5;0-5;11	40	64.00	24. 9	± 3.90	67.94 60.06

TABLE XIV

PROBABLE TRUE MEANS WITHIN LIMITS OF ± 3 S.D.

Age Group	No. Cases	Obtained Means	S.D. of Means	Probable True Mean
3;0-3;11	13	10.96	$\pm 3 (1.91)$	16.69 5.22
4;0-4;11	33	36.75	$\pm 3 (3.28)$	46.75 26.91
5;0-5;11	40	64.00	$\pm 3 (3.94)$	75.82 52.18

(b) Reliability of Difference between Means.- Sampling reliability of the difference between the means was determined for four- and five-year groups and for three- and four-year groups. Table XV gives data for these calculations and range within which probable true differences lie. Thus the difference between the means for the two older groups is 27.25 and the standard error of this difference is 5.09. Practical certainty locates the probable true difference within the limits of 42.52 and 11.98, and by the general rule, the difference is shown as significant. It follows,

TABLE XV

SAMPLING RELIABILITY OF DIFFERENCES BETWEEN OBTAINED
MEANS IN TERMS OF STANDARD DEVIATION

Age Groups Compared	Means	Diff. bet. Means	S.D. Diff.	Range of Probable True Difference 68-100 chances	3 S.D. Diff.	Range of Probable True Difference 99-100 chances
Five -	64.00	27.25	± 5.09	32.24-	± 15.27	42.52-
Four	36.75			22.16		11.98
Four -	36.75	25.79	± 3.39	29.18-	± 10.17	35.96-
Three	10.96			22.40		15.52

then, that the obtained difference between the means for four- and five-year groups is not only reliable but 11.98 points or 2.3 S.D., or 98.9 per cent greater than necessary to insure a difference above zero. By similar determination the standard error of the difference between the means of the three- and four-year groups is 53 per cent greater than the amount necessary to insure reliability.

The foregoing study of reliability of means and differences between means supports the assumption that, under the limitations set forth above, an analysis of number ability on the basis of obtained measures is reasonably valid.

Growth in Number Ability of Comparable Age-Groups

Analysis was made (1) of measures of central tendency to determine rate and uniformity of growth; (2) of measures of variability to define absolute and relative deviations; and (3) of obtained measures in relation to general intelligence.

(1) Rate of Growth.- Median scores for comparable groups, Table XII, show that performance improves with regularity from age to age. The median scores are as follows:

3;0-3;11 9.9
4;0-4;11 32.5
5;0-5;11 58.0

The increment between third and fourth and that between fourth and fifth is approximately uniform—22.6 in the former, and 25.5 in

the latter. Rate of growth, as measured by median scores, yields a relatively straight line development for the age levels within the limits of this investigation. Contrary to growth curves for physiological development, growth is evenly spaced and not characterized by sudden variations in rate. The number of cases is limited, especially at three-year level. Whether a larger number would show variations or even spurts analogous to those of early physical development, the evidence within the investigation is not adequate to establish.

Comparison of the mean for a given age group with that of the preceding year defines increment of mean growth. If the size of the increment in successive years remains relatively constant, growth curves extend in a parallel direction. On the other hand, increase in size of yearly increment tends toward divergence, and decrease or decrement, tends toward convergence of growth curves.¹

Table XII presents the means and these are shown graphically in Figure 4 by dotted lines. Mean performance advances at successive age levels. Table XV gives the differences between means, or the increments of growth; the increment for younger groups is 25.79 and that for older is 27.25. The latter is 1.46 points greater, an amount less than the standard errors of the means and hence not significant, though probably due to peculiarity in sampling. With this exception, the growth curve, plotted from data for the means and represented in Figure 5 by the full drawn line, is a straight line.

The analysis of growth in number ability as measured by the increments of the medians and means at successive age-levels indicates that yearly rate is relatively constant with the exception of the slight deviation noted.

(2) Variability of Growth.— Standard deviations for the three age-groups (Table XII) increase from 6.9 in three-, to 18.8 in four-, to 24.9 in five-year groups. These sigmas are graphically shown in the hatched portions of the histograms of Figure 4. From the latter it is apparent that ability for three year olds is limited in amount and closely centered about the mean; and that for four and five year olds the "spread" is much greater. The increase at four year level is approximately twelve points (11.95) and at five, practically one half as much (6.05). In other words, increase in variability within each group is proportionate but not uniform.

The scores of individual children (Tables XVIII-XX, Chapter

¹Frank N. Freeman, Mental Tests, p. 349 ff. Boston: Houghton Mifflin Co., 1926.

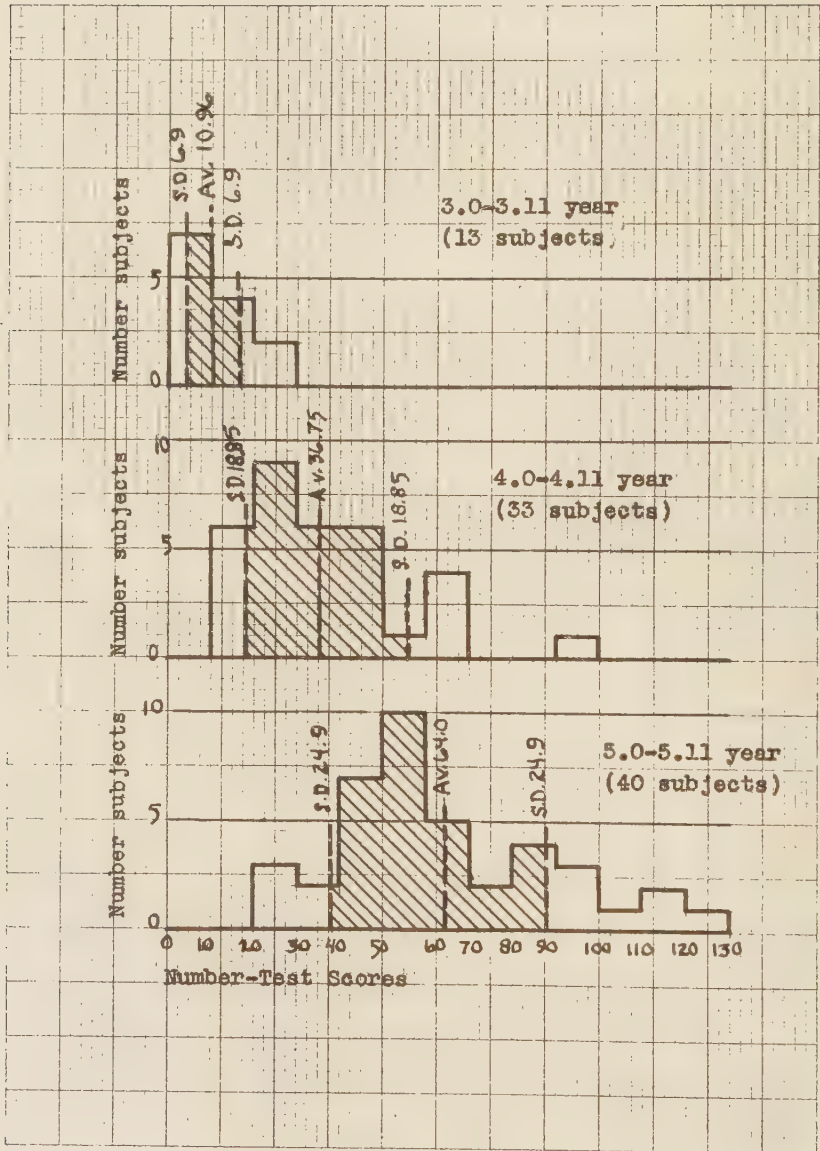


Fig. 4.- Comparison of Number-Test Scores of Comparable 3-6 year-groups (86 subjects), Showing Means and S.D.

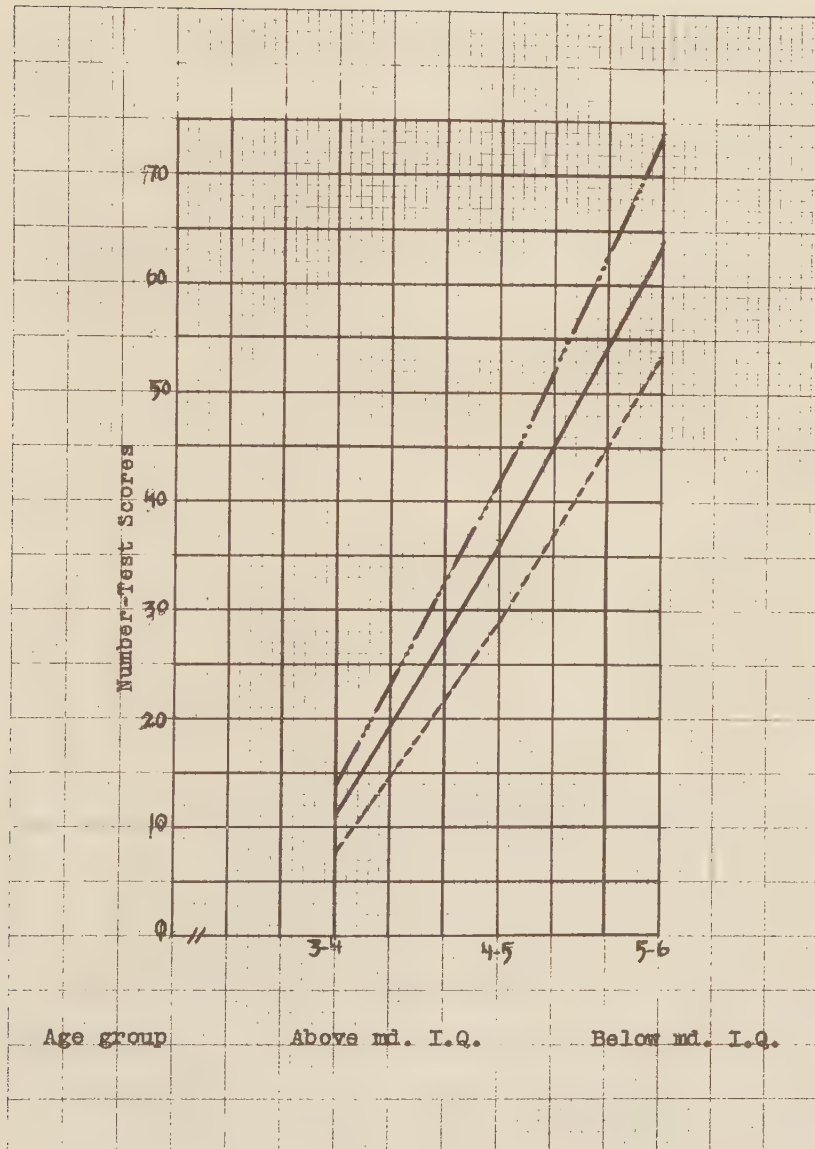


Fig. 5.- Growth Curves for Number Ability as shown by Means.

IV) offer explanation. Test 5 shows that twenty-five children, or 62.5 per cent of the oldest group, "broke" the test and fourteen or 35.0 per cent made a similar record in Test 6. In other tests, notably 9 and 13, children proficient in rational counting made very high or perfect scores. Table XIX discloses a similar situation among a smaller number of children in the four-year group. For such children as "broke" the test, range of difficulty was insufficient to give a complete measure of their variability, hence, the standard deviation would be, therefore, less because of sampling error in selection of test items.

These data support the inference that in certain, perhaps narrow aspects of general number ability, individuals approach the asymptote of the curves for these aspects (cf. Figures 12, 13, and 16, Chapter IV) though general ability advances at a fairly regular pace. Freeman explains such advance as:

"not so much from one ability to another as from the ability to perform a task at one level to the ability to perform a task at another level, all levels requiring the same general kind of ability."¹

Number Ability in Relation to General Intelligence

The last problem of the chapter seeks to determine to what extent general intelligence influences development of number ability. Comparing attainment of higher and lower intelligence groups by the same technique as that used with entire age-groups shows that: (1) medians for the former are greater than for the latter, as indicated in Table XII by encircled figures; (2) means, given in the same table and plotted in growth curves shown in Figure 5, indicate attainment at distinctly separate levels for age-groups differently weighted with respect to the factor of general intelligence; (3) sigma values are greater for higher than for lower intelligence groups; and finally (4) growth curves, Figure 5, appear to diverge somewhat at fourth and fifth-year levels.

A possible explanation for the divergence in growth curves is as follows: In the preceding discussion of this characteristic in growth curve for entire age groups, it was found that increase in increment for fifth and to a lesser degree for fourth year was largely a function of perfect scores and that this superior attainment, in practically all instances, was accompanied by proficiency in rational counting.² It is possible and very probable that

¹Frank N. Freeman, op. cit., p. 343.

²Cf. scores in Tables XVIII-XX.

control of this process is the causal factor for the spurt in the growth curve in fifth and for the slight but perceptible increase at fourth year level. The causal factor, in other words, appears to be mastery of the idea of correspondence in counting. Stern,¹ on the other hand, reports an acceleration for fourth instead of fifth year and attributes it to mastery of the cardinal number three, which he says, "clears the way for higher numbers as well." Further discussion of this factor is deferred until the detailed analyses of the processes of counting and group recognition are presented in the following chapter.

Comparison of sigmas for groups above and below median I.Q. supports the conclusions reached in the corresponding study of entire age groups, namely, that variability is evident at all age levels, but that the size or amount of variability deviates from constancy. Table XVI gives data for all groups.

TABLE XVI

DIFFERENCE BETWEEN SIGMAS OF GROUPS ABOVE AND BELOW MEDIAN I.Q.

Age Group	Above Median I.Q.		Below Median I.Q.	
	S.D.	Difference	S.D.	Difference
3;0-3;11	6.85		5.0	
4;0-4;11	21.35	14.50	13.55	8.55
5;0-5;11	23.37	2.02	22.20	8.65

The table shows that the amount of variability between the two youngest "bright" groups exceeds that between the corresponding less bright groups. The "spread" for the former is 69.5 per cent greater and much less evenly spaced, and indicates that ability among higher intelligence group is less stable and uniform. Subjects vary from one another to a much greater degree than do subjects in the like age group below median I.Q. In this latter group ability is not unlike that at third year level, viz., comparatively limited in amount and restricted to a narrow range. Figures 6 and 7 graphically present the foregoing data.

The difference (2.02) in variability between the two older "bright" groups is much less than that for corresponding groups below the median. This may be explained as due to the same factor noted in the preceding discussion, viz., the lack of discriminative

¹William Stern, op. cit., p. 416.

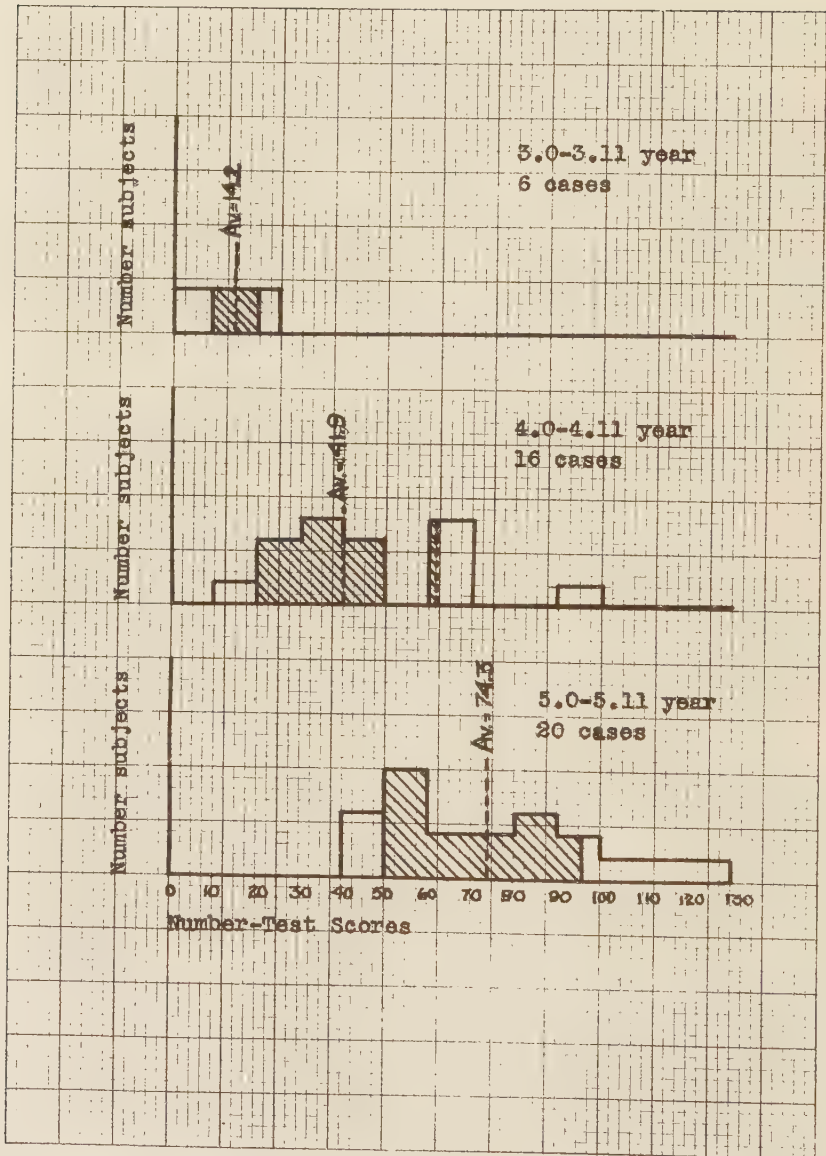


Fig. 6.- Comparison Number Test Scores for 3-6 Year Groups Above Median I.Q.-110.

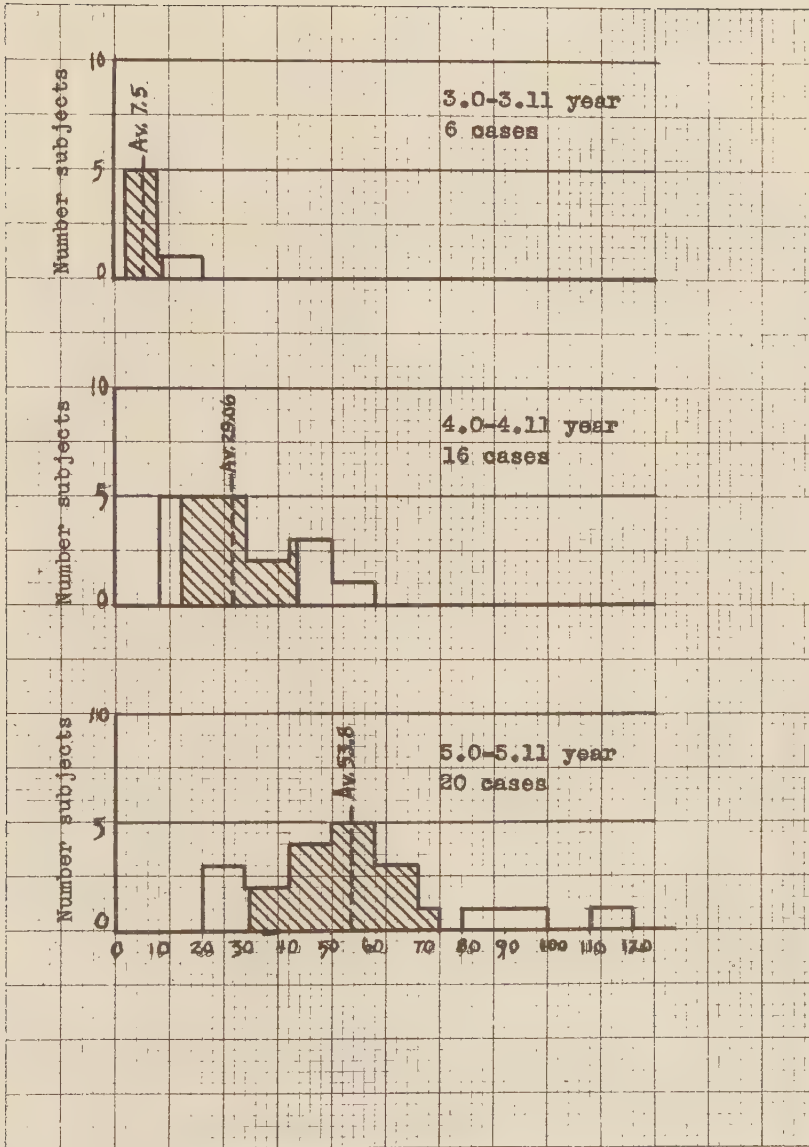


Fig. 7.- Comparison of Number Test Scores 3-6 Year Groups Below Median I.Q.-110.

test-difficulty for the oldest and brightest children.

Though the numbers are small the general indication seems to be that greater variability is characteristic of higher mental groups who appear to grow at an accelerated pace in comparison with corresponding less talented groups.

Summary and Conclusions

To determine the relation between growth in number ability and general intelligence for ages three to six has been the purpose of the chapter. For purposes of the problem, three age-groups comparable in general intelligence but different in population were selected from the 125 subjects described in Chapter I. The common median intelligence quotient was 110, thus characterizing the groups as slightly above average intelligence. Comparability was secured at a sacrifice in number from 125 to 86 distributed as follows: 40 in fifth, 33 in fourth, and 13 in third year. The population of the two older groups met requirements for statistical adequacy but, although that of the younger group failed to do so, it was included, nevertheless, because of diagnostic value in analysis of trends determined for older groups. This procedure thus limited the problem to groups of the given ages ranking slightly above average intelligence.

Year-by-year growth was first analyzed for age-groups as wholes and then for age groups above and below the common median intelligence quotient. A consideration of obtained means for the number-test yielded a measure of age-progress and difference between means at successive age levels, a measure of the degree of uniformity in growth. Standard deviations indicated the variability at successive age-levels; and differences between sigmas evidenced the size of deviations in variability within age-groups. The reliability of the means and their differences, determined before employing them in the manner just noted, indicated that these measures, by the usual test, were not only significant but considerably exceeded statistical requirements.

Results of the foregoing analysis, within limitations previously noted,—viz., insufficiency of population in three-year group and slight superiority of intelligence, support inferences included in appropriate discussions and brought together at this point for summary emphasis.

Age progress in functions measured by the test was shown by the means and medians to be continuous and distinctly differentiated. Rate, as measured by yearly increment in mean and median performance, was relatively uniform with a tendency to increase somewhat in the oldest group. The slight spurt in development

contrary to findings of other investigators, was found at fifth instead of at fourth year level and was accompanied by proficiency in rational counting.

Achievement for superior intelligence groups appeared at distinctly higher levels and curves for these groups maintained a fairly constant relation to those of lower intelligence. Acceleration in rate appeared at fifth and in less degree at fourth year levels, but greater in amount for higher than for age-groups of lower intelligence.

Variability increased from year to year but the increment was not constant owing to inadequacy of difficulty range in test items. Variability was greater for higher than for lower intelligence groups.

A discussion of growth in the three component processes—counting, recognition of aggregates, and combining aggregates—follows in Chapter IV.

CHAPTER IV

DEVELOPMENT OF THREE PROCESSES: COUNTING, RECOGNIZING AGGREGATES, AND COMBINING AGGREGATES

Purpose and Method of Analysis

The preceding chapter concerned itself with development of number ability in relation to factors of age and general intelligence. The present chapter analyzes number ability in relation to development of component processes: counting, recognizing of aggregates, and combining aggregates. Answers to such questions as the following are sought. Does acquisition of ordinal precede, parallel, or follow that of cardinal or group number? What are characteristics of early and later stages of growth in acquisition of both ordinals and cardinals? To what extent is ability in combining aggregates dependent upon counting and group recognition? In other words, the chapter purposes to characterize stages in the development of these three component factors and to determine to what extent growth is interdependent.

Inasmuch as the children used were the three-to-six comparable age-groups described in the preceding chapter, an analysis of their achievement afforded a basis for determining characteristics of year-by-year progress. Analysis of age-group achievement was supplemented by that of individual achievement selected from successive age levels on the basis of pertinency to the problem.

It will be recalled that tests were grouped into three series as follows: Series I, Subtests 1-3 dealing with counting; Series II, Subtests 4-8, with group recognition; and Series III, Subtests 9-12, with combining two aggregates and Subtests 13-16, with combining three aggregates. The distribution of average scores in each subtest for all age-groups appears in Table XVII. Scores of entire groups are shown in the last column for each year and those for higher and lower intelligence groups in the two columns at the left respectively.

Individual scores for all eighty-six children are given in Tables XVIII-XX. Other data employed in the analysis of individual achievement include Teachers' Ratings¹ and response notations

¹See Appendix, pp. 151-57.

TABLE XVII

SUBTEST SCORES FOR HIGHER AND LOWER AND COMPARABLE
INTELLIGENCE AGE-GROUPS

Test No.	Ages 3-0 to 3-11			Ages 4-0 to 4-11			Ages 5-0 to 5-11		
	Low I.Q. Group	High I.Q. Group	All	Low I.Q. Group	High I.Q. Group	All	Low I.Q. Group	High I.Q. Group	All
1	.30	.58	.45	1.20	2.40	1.76	2.37	4.32	3.34
2	.32	.53	.44	1.03	1.91	1.45	2.16	3.45	2.82
3	.00	.00	.00	.00	1.56	.75	4.40	6.55	5.47
4	1.16	3.50	2.38	2.69	4.38	3.57	4.15	5.50	4.82
5	1.00	2.66	2.23	5.69	6.12	5.87	6.00	6.60	6.30
6	1.16	2.33	1.61	3.25	5.25	4.27	5.30	6.10	5.70
7	1.50	1.50	1.53	2.25	2.50	2.36	3.10	3.85	3.47
8	.33	.83	.61	2.25	2.68	2.51	3.80	4.85	4.33
9	.50	.83	.61	3.31	5.68	4.48	6.95	6.95	6.95
10	.16	.50	.30	1.88	3.06	2.45	4.25	5.75	5.00
11	.00	.66	.30	1.13	2.75	1.90	3.40	4.85	4.12
12	.00	.16	.07	.94	2.31	1.57	2.45	5.10	3.78
13	.16	.00	.07	1.31	2.81	2.00	2.65	3.90	3.27
14	.00	.00	.00	.75	.93	.81	1.05	2.80	1.92
15	.00	.00	.00	.63	.62	.60	1.00	1.80	1.40
16	.00	.00	.00	.38	.68	.51	.85	1.55	1.20
Total . .	6.59	14.08	10.60	28.69	45.64	36.86	53.88	73.92	63.89

entered on test-forms during administration of the test. Study of the latter data disclosed methods and difficulties characteristic of various ages, and study of the former supplemented findings relative to individual differences.

Counting Ability at Successive Age Levels

Growth curves for counting, plotted from data given in Table XVII, are presented in Figures 8, 9, and 10. It should be recalled that scores for Subtests 1 and 2 were derived by dividing the number successfully counted, without intervening error, by 10; thus, a score of 3.34 indicated that the average number counted was 33.4. To facilitate comparison of development at different

TABLE XVIII
SUBTEST SCORES FOR 5.0-5.11 COMPARABLE AGE-GROUP ACCORDING TO I.Q. RANK (40 subjects)

Subjects	I.Q.	Subtests														16
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	138	10.0	7.0	20	5	7	6	9	6	8	10	7	8	3	4	4
37	133	6.0	6.7	5	6	7	7	3	3	8	2	1	2	4	5	2
18	132	2.6	3.9	5	7	7	6	5	6	6	8	4	5	5	1	0
36	131	3.7	3.9	5	7	7	6	0	4	7	6	4	4	3	4	2
23	130	1.0	1.1	0	3	7	7	4	6	9	9	7	9	3	4	1
4	122	10.0	10.0	20	7	7	7	3	6	8	4	7	8	3	4	5
45	122	3.9	3.9	10	7	7	7	7	6	5	6	7	7	5	4	2
14	121	10.0	5.0	20	3	7	7	3	6	9	6	7	8	3	4	2
26	120	1.3	1.3	0	6	7	6	7	4	9	10	2	4	5	2	3
13	119	3.9	3.7	10	7	7	7	3	6	9	6	9	7	4	5	4
8	118	10.0	3.8	5	6	7	7	3	6	9	8	9	8	5	4	0
2	118	3.9	2.9	0	7	7	7	3	5	9	3	9	7	4	1	3
33	118	2.9	2.7	0	7	7	7	3	7	6	3	5	4	5	1	1
15	115	1.7	1.7	0	5	5	5	2	4	7	2	0	3	2	1	2
5	113	1.2	1.2	5	5	7	7	5	1	3	7	6	5	3	1	0
7	113	1.8	1.8	5	5	7	7	5	4	6	3	3	5	3	0	4
50	112	3.9	2.9	5	5	7	7	3	4	2	4	1	1	4	2	2
48	112	1.2	1.0	0	5	7	3	5	7	7	6	6	5	4	4	0
46	111	1.5	1.5	11	5	7	5	6	6	9	6	2	8	5	2	0
38	110	5.9	3.9	5	5	6	6	3	4	7	7	6	4	5	0	4
12	109	2.9	3.6	10	7	4	4	5	6	10	1	2	0	4	2	0
29	109	2.1	3.9	5	2	6	6	3	4	9	8	5	0	5	0	5
47	107	1.4	1.4	10	2	6	6	4	3	10	2	9	8	4	3	0
17	106	10.0	6.9	20	7	7	6	4	5	9	5	1	0	4	0	2
30	106	1.4	1.3	5	5	7	6	3	2	8	5	5	1	5	0	2
16	106	2.9	1.4	0	2	7	3	4	0	9	1	4	0	4	2	0
44	104	3.9	2.9	5	3	5	4	3	5	1	8	6	2	0	1	0
40	103	.8	.8	0	6	6	4	6	7	10	6	1	2	4	0	0
43	103	2.0	1.4	0	2	6	4	3	3	9	5	4	2	0	2	0
49	102	1.1	1.1	0	7	7	5	6	3	7	1	7	0	5	0	0

TABLE XVII (Continued)

Subjects	I.Q.	Subtests															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
34	102	1.1	1.1	0	2	4	4	2	3	3	2	1	2	2	0	0	1
32	100	1.3	1.3	5	1	6	5	2	4	8	4	2	4	0	0	0	0
35	100	1.4	1.3	0	6	5	6	1	4	8	1	4	1	2	0	1	0
24	98	2.8	1.9	0	2	6	5	3	3	2	1	1	1	2	0	0	0
27	97	1.0	1.0	5	4	7	7	3	4	9	4	2	3	0	2	2	0
19	91	1.3	1.3	5	4	5	6	2	7	9	6	4	2	5	0	3	3
42	90	2.9	2.9	5	6	7	7	4	6	9	9	8	9	5	3	3	5
21	89	4.0	4.9	5	6	7	6	3	3	10	2	2	1	5	2	2	1
39	89	1.4	1.2	5	7	6	7	6	5	8	9	2	0	5	1	0	0
25	82	1.6	1.6	0	2	7	5	2	4	1	1	1	3	0	1	0	0
Total		133.8	113.1	219	193	249	228	139	174	278	200	165	152	131	77	56	48
Mean		3.35	2.83	5.48	4.82	6.30	5.70	3.47	4.33	6.95	5.0	4.12	3.78	3.27	1.22	1.40	1.20
Median		2.1	1.9	5.0	5.0	7.0	6.0	3.0	4.0	8.0	6.0	4.0	3.0	4.0	2.0	2.0	0.0

, 51 ,

TABLE XIX

SUBTEST SCORES FOR 4.0-4.11 COMPARABLE AGE GROUP ACCORDING TO I.Q. RANK (33 Subjects)

Subjects	I.Q.	Subtests															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
51	141	2.0	1.3	5	7	6	6	4	5	7	6	7	4	1	2	3	1
53	133	3.8	3.8	5	2	5	4	2	2	9	5	5	4	4	3	3	2
84	132	1.4	1.4	0	6	7	7	3	3	10	1	2	1	5	0	0	0
98	128	7.9	2.9	5	6	7	7	2	3	4	5	3	4	5	0	0	0
60	125	4.9	4.9	5	7	7	7	5	5	9	7	7	7	5	4	3	5

TABLE XIX (Continued)

Subjects	I.Q.	Subtests															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
87	123	1.3	1.3	0	4	6	7	2	2	9	5	5	3	2	0	0	0
78	122	1.6	1.6	0	5	7	6	2	3	5	1	1	1	4	0	0	0
95	122	2.0	1.9	0	3	5	3	1	3	5	1	1	3	0	0	0	0
67	120	3.9	3.9	5	4	4	7	6	2	5	1	1	3	0	1	0	0
65	120	1.4	1.0	0	4	7	2	1	2	5	1	1	3	0	0	0	0
100	119	1.2	1.2	0	3	4	3	6	2	5	1	1	3	0	0	0	0
56	114	1.8	1.4	0	7	7	7	3	2	9	1	1	3	0	0	0	0
66	114	1.3	.9	0	5	7	5	3	0	8	1	1	3	0	0	0	0
81	114	1.1	.8	0	2	6	4	1	1	0	1	1	3	0	0	0	0
96	114	1.0	.9	0	1	6	2	1	4	0	2	1	3	0	0	0	0
80	113	1.8	1.5	0	3	7	7	1	4	8	1	1	3	0	0	0	0
93	111	.6	.8	0	5	5	5	2	4	4	1	1	3	0	0	0	0
68	109	1.2	.9	0	3	7	2	2	4	9	1	1	3	0	0	0	0
58	108	1.2	2.2	0	4	7	6	2	4	0	1	3	2	4	0	0	0
82	107	.8	.3	0	1	6	3	1	0	1	6	0	0	0	0	0	0
92	106	1.2	.8	0	1	5	2	1	3	0	1	0	0	0	0	0	0
59	105	1.1	1.1	0	2	3	2	1	0	1	2	0	0	0	0	0	0
85	100	.6	.7	0	2	4	3	3	2	0	6	1	0	2	0	0	0
54	98	1.2	1.2	0	6	7	4	4	3	1	2	2	1	0	2	0	0
77	98	1.5	1.0	0	3	7	7	2	2	10	1	1	1	0	2	0	0
63	96	1.8	1.4	0	1	7	4	3	2	1	2	1	1	0	2	0	0
62	95	1.4	1.4	0	3	6	3	0	1	1	2	1	1	0	2	0	0
74	95	1.0	1.4	0	1	7	4	3	1	1	2	1	1	0	2	0	0
71	92	1.2	1.2	0	5	0	3	0	3	1	2	1	1	0	2	0	0
57	90	1.8	1.3	0	2	7	3	2	3	8	1	1	1	0	2	0	0
72	88	1.1	1.3	0	4	5	0	2	3	2	1	1	1	0	2	0	0
73	86	1.0	1.2	0	2	7	3	4	3	1	2	1	1	0	2	0	0
61	81	1.2	.7	0	1	6	3	3	1	1	1	1	0	0	0	0	0
Total		58.3	48.1	25	118	194	141	78	83	148	81	63	52	66	27	20	17
Mean		1.77	1.46	.76	3.58	5.88	4.27	2.36	2.52	4.48	2.45	1.91	1.58	2.0	.8	.60	.51
Median		1.2	1.1	0.0	3.0	6.0	3.0	2.0	2.0	3.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0

TABLE XX

SUBTEST SCORES FOR 3.0-3.11 COMPARABLE AGE GROUP ACCORDING TO I.Q. RANK (13 Subjects)

[illegible]

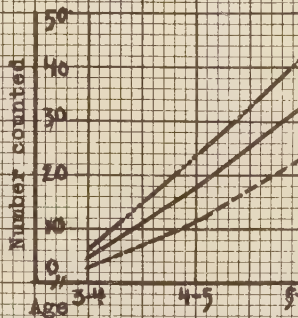


Fig. 8.- Rote Counting

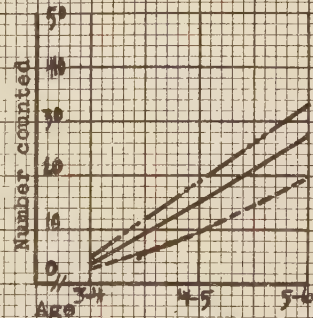


Fig. 9.- Rational Counting

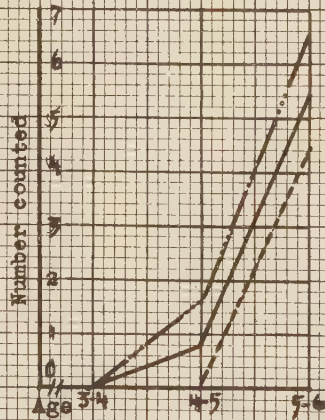


Fig. 10.- Counting Backwards

Age group — Above md. — Below md. ----

Figures 8, 9, 10.- Showing mean achievement in Rote, Rational and Backward Counting for 3-6 Comparable Age-Groups and for Corresponding Age Groups Above and Below Median I.Q.

age-levels, the actual average number counted by each group was used for the ordinates of the foregoing graphs instead of the average score. Full drawn lines indicate curves for entire age-groups, broken lines those for higher, and dotted lines those for corresponding lower intelligence groups.

Growth curves for entire age-groups show appreciable progress in all phases of counting. In rote counting the range is from 4.5 in the three-year group to 33.4 in the five-year. The attainment in the fourth year is 2.91 times that of the third and in the fifth year 6.42 times that of the third year. This advance, subsequent analysis of individual records discloses, is due to the emergence of greater control of numeral terms. A similar test at the Child Welfare Station, University of Iowa,¹ given to nursery school and kindergarten children reports somewhat lower averages for the successive age-groups from three to six, as follows: 3.0, 10.6, and 23.7. The higher averages in the present study may be explained on the basis of selection—the age-groups are of comparable and above-normal intelligence. The Iowa study does not use groups equated in intelligence.

An examination of individual records of three-year olds, Table XX, shows that all children counted by rote to 2 and approximately two-thirds (69.2 per cent) counted to 4. One child reached 12. Subjects 122 and 110 counted in random order beyond 2. They have learned number names but have not as yet fixed with certainty their correct serial order. On the other hand, Subjects 101 and 119 have advanced much further in control of serial order.

An analysis of individual records for four-year olds, Table XIX, offers explanation of greater achievement as well as increase in variability at this level. In the three-year group one child reached 12; in the four-year group more than one half (51 per cent) surpassed this record, almost a fifth (18 per cent) bridged the second decade, and approximately one-eighth (12 per cent) counted beyond this point. On the other hand, three subjects (9 per cent) failed to go beyond 8, an achievement inferior to that of the best three-year subjects. Ability in the third year is small in amount and confined within a narrow range; that in the fourth year is not only greater in amount but spread is markedly wider, thus indicating increased variability in the function measured.

Rote counting is more conspicuously developed in fifth, Table XVIII, than in third or fourth years. Twenty-two of the forty subjects (55 per cent) bridged the second decade; of these

¹Bird T. Baldwin and Lorle Stecher, op. cit., pp. 152-169.

nine (22.5 per cent) counted into decades between 30 and 60 and five (12.5 per cent) reached 100. A lesser achievement was made by fifteen subjects (37.5 per cent) who bridged the first decade but failed to reach the second. One subject (2.5 per cent) did not progress beyond 8, a score surpassed by two subjects in third and by all but four subjects in fourth year. Though the average, 33.4, is high for five year children, one child is still engrossed with the problem of mastering serial order of numerals.

This record, as did that of the three- and four-year groups previously analyzed, surpasses that of the Iowa group. It furthermore compares favorably with data for six-year school entrants examined by Buckingham and MacLatchy.¹ These investigators report a median of 27.0 (mean not given) for 1290 pupils and add that 90 per cent of this total number counted to 10, 60 per cent to 20, and 12 per cent to 100. The superior achievement of the five-year group in the present investigation is patent and explicable in terms of the selective factor of intelligence as aforementioned. A second factor may be that of the composition of the two groups; the six-year school entrants included many from small villages and rural districts while the five-year pupils of the present study were all city children.

The averages for rational counting at the three successive ages are: 4.4, 14.5, and 28.2. The advance for the fourth year is 2.29 times and for fifth year 5.4 times that at third year level. A comparison of these averages with those for rote counting discloses that, with the negligible exception of the youngest group, ability in rational counting is but slightly less than that of rote counting. Earlier investigators reported superior attainment in counting objects.² The foregoing finding in the present investigation is in general agreement with that reported for pre-school pupils³ and school entrants.⁴

The lesser achievement, though small, is explained for children of the early ages herein studied, by the greater complex-

¹B. R. Buckingham and J. MacLatchy, op. cit., pp. 491-92.

²Albert Duncan Yocum, An Inquiry into the Teaching of Addition and Subtraction, pp. 13-14. Philadelphia: Avil Printing Co., 1901. In a group of 50 boys and 50 girls whose average age was less than six years, 20% counted by rote beyond 20; but 32% of the boys and 38% of the girls counted with objects beyond 20.

³Bird T. Baldwin and Lorle Stecher, op cit., pp. 152-169.

⁴B. R. Buckingham and J. MacLatchy, op. cit., p. 495.

ity of the process involved in enumerating objects in a series.¹ Rote counting employs simple recall of verbal numerals in correct serial order, but rational counting uses this function and in addition requires the synchronizing of the correct verbal numerals with the simultaneous discrimination of corresponding items in an objective series.²

Summary of Development in Three Types of Counting.—The counting processes within the limits of this investigation have been shown to involve complex mental activities. With least mature subjects the first reactions to an objective series results in the discrimination, in some primitive manner, of one or more items. Youngest subjects, those included in the preliminary test group—Class One, described in Chapter II—were able to take this step. One subject twenty-seven months old discriminated the objective items by pointing and saying, "dee, dee, dee"; three other subjects a few months older used the terms, "dee-one, dee-one, dee-one," or "dat-one, dat-one," or "one, one, one."

This primitive form of reaction is replaced later, as was shown in the study of three-year subjects, by the first few terms of the subjective series. A few had learned to use 1, 2 and others 1, 2, 3, 4; one subject had bridged the first decade and reached 12.

By the end of the fourth year more than half the group had mastered rote counting to 10; approximately a fifth of the higher intelligence group counted into the decades beyond 20. In the fifth year all subjects except one mastered rote counting to 10,

¹Charles H. Judd, Psychological Analysis of the Fundamentals of Arithmetic. Supplementary Educational Monographs, No. 32. Chicago: Department of Education, University of Chicago, 1927. In discussing the factor of correspondence, Judd says in part, "It is a rational form of consciousness and, as such, can be thought of as a mental pattern which unites the two factors, number names and discriminated objects, holding these two factors together in a certain form of organized experience."—p. 70.

²Josiah Royce, "The World and the Individual," Supplementary Essay: The One, the Many and the Infinite, (First Series: Gifford Lectures). New York: Macmillan Co., 1901. Royce points out that the series idea is the result of iterative, recurrent thinking that applies the definite first unit to successive multiple number, but always the relationship, discovered by reflective thinking, is simultaneously held in mind. "Such processes of passing to 'one more' instance of a given type, are processes not of barren repetition, but of genuine progress to higher stages of knowledge."—p. 529.

and approximately half the group accomplished the same feat to 20, and twelve and a half per cent reached 100.

In rational counting as shown by Tables XVIII and XIX more than half the four-year (54.5 per cent) and a few less than half the five-year group scored lower in rational than in rote counting. For these children the former process was more difficult than the latter. The difficulties are equally balanced for approximately a third more in each group whose scores are the same in both types of counting. It is noteworthy that a few in each group had slightly superior scores in rational counting. The range at three-year level is so narrow that differences are not significant. With youngest children the difficulties attendant upon inability to grasp the idea of correspondence limits rational counting to approximately the first decade. But 61 per cent of the group were able to reach 4, while one child counted beyond to 11. Two-thirds of the fourth and all the fifth, except one, bridged the first decade; 15.1 per cent of the former and 47.5 of the latter counted beyond the second decade; a few counted as far as 70 or 100. Ability of this latter type is indicative of a maturing control of the counting process.

Ability in counting backwards does not appear before the fourth year, and then, in the higher intelligence group only; five subjects (15 per cent) counted backwards from 5; in every instance the achievement was accompanied by high scores in rote and rational counting. Though ability in counting backwards develops somewhat more in fifth year, it does not exceed 6.55 even in the group of higher intelligence. Complete control of verbal series has not been mastered sufficiently to give facility in backward counting beyond a few initial numbers. The difficulty arises from the fact that associations set by habituation in forward counting must be reversed in serial order.¹

Growth curves for higher and lower intelligence groups (Figures 8, 9, and 10) show that superior attainment accompanies superior intelligence. Higher groups at each age level approximately double the scores of corresponding lower groups in rote and in rational counting.

Rational counting is more than repeating a series of numerical terms in an established order; it involves the concept of correspondence between these and the items of an objective series. That subjects memorize terms of the subjective series

¹Hulsey Cason, "Specific Serial Learning; A Study of Backward Association," Journal of Experimental Psychology, IX (1926), 195-227; 299-324.

without relating them to the objective series is evidenced by lack of skill in rational counting. Most of the children tested, however, steadily advanced from three to six years in ability to match the items of the two series, though until the first three or four decades are reached, achievement in rational counting is less than that in rote counting.

Counting backwards, as is shown in Figure 10, is a difficult process for three to six year children. None of the three year, and none of the four year group below median intelligence are able to reverse the order of the subjective series. One year later in the fifth year, approximately one-third of the group accomplish the task from 5, one-tenth from 20. Backward counting was an entirely new idea to many subjects.

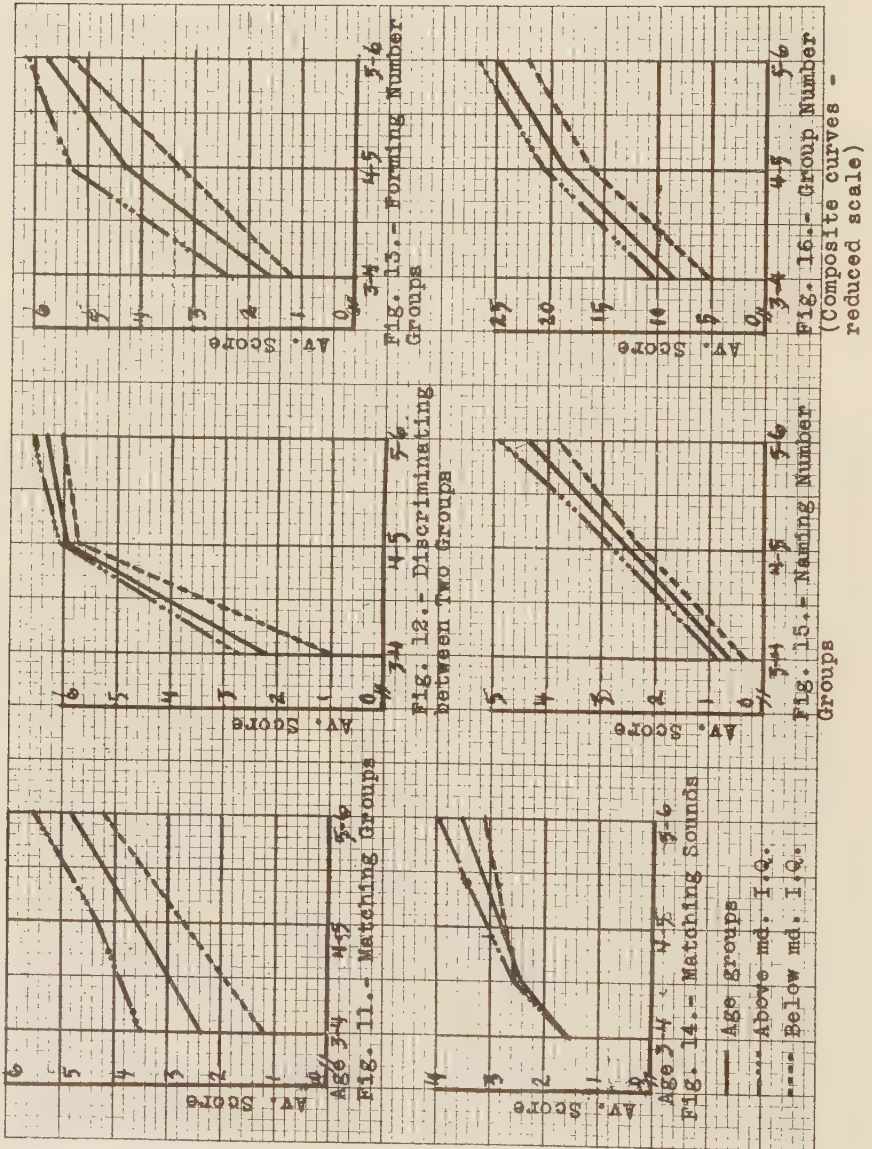
Recognizing Group or Cardinal Number

Five tests were used to determine to what extent recognition of group or cardinal number is developed before the age of six years. Each test measured one element of recognition as follows: matching visible group number with like amounts, Subtest 4; discriminating larger from slightly smaller groups, Subtest 5; forming groups upon oral request, Subtest 6; matching grouped sounds with like sound groups, Subtest 7; and naming group number in timed exposure, Subtest 8.

Growth Curves for Comparable Age Groups.- Average scores for each age group appear in Table XVII. Growth curves for separate subtests are shown in Figures 11-15, and that for the five tests considered together in Figure 16.

Though curves for separate subtests indicate growth in each of the five elements at successive age levels it is manifest that growth in some elements is greater than in others. In the third year, for example, attainment is lower for discriminating and forming (Subtests 5 and 6) than for matching visible aggregates (Subtest 4); but in the fourth and fifth years the situation is reversed. Achievement in discriminating and forming number in these older groups not only surpasses that of matching visible number groups but also that of naming aggregates (Subtest 8) and that of matching grouped sounds (Subtest 7). Furthermore, rate of growth is more rapid in fourth than in fifth years for both functions. The marked spurt in discriminating and forming number is due in large measure, as pointed out previously,¹ to high percentage of perfect scores. In Subtest 5, 48.4 per cent in fourth year and 62.5 per cent in fifth year "broke" the test. The corresponding

¹Cf. Chapter III, page 42.



per cents for Test 6 are 24.2 and 35.0.

The explanation of high scores may be found in certain characteristics inherent in the subtests themselves. Subtest 5 required discriminating between two aggregates that differed from each other by one unity. The aggregates were represented by a grouping of dots in varied pattern arrangement on separate cards. Duplicate cards showing aggregates of the same size followed the same pattern. It is possible that in this test responses were made to spatial factors of aggregates rather than to number of unities composing them. In a testing situation somewhat similar to this Binet¹ reports that a four year old child could distinguish correctly in nine out of ten trials the larger of two groups of counters. A few children who experienced difficulty in counting gave evidence of responding to spatial characteristics in the matching tasks of Test 4; they enclosed areas similar in size to areas covered by aggregates that they were attempting to match and then filled in the space with a handful of objects. The spatial extent of the aggregate is perceived but not the number of unities composing it.²

Similarly in Subtest 6, a characteristic of the test situation itself aids interpretation. In the act of forming an aggregate in response to the test, subjects have opportunity to count the objects as they place them in position. While counting ability at three year level is not far advanced as was shown in the preceding section of this chapter, it develops rapidly in the fourth and fifth years with grasp of the one-to-one relation. Hence, forming number groups improves rapidly in fourth year and approaches a perfect score in fifth year because all subjects at this age not only have gained some proficiency in counting, but have developed ability in applying counting methods in forming aggregates included within limits of the test.

Achievement in recognition of aggregates is lowest in matching grouped sounds, Subtest 7. The average for three year

¹Alfred Binet, "La Perception des Longueurs et des nombres," Revue Philosophique, XXX (1890), 68-81. The groups numbered 15 and 18, 16 and 18, 17 and 18. When counters were of the same size and color judgments were based upon spatial characteristics rather than upon specific unities composing them. [Cf. MacLachy, Childhood Education, VII (October, 1930), 59.]

²Koffka, Kurt, Growth of the Mind, pp. 332-35. Harcourt, Brace & Co., 1925. This ability K. terms "prenumerical constructs" and maintains, with Wertheimer that "the formation of natural groups and conglomerate constructs is prior to counting."

children is less than 2, for four year olds less than 3, and for five year olds less than 4. Focusing attention upon a succession of sounds, identifying the number of them and reproducing a like number were tasks that made exacting demands upon young children. Many failed entirely, others reproduced 2 or even 3, and a very few reproduced all numbers from 2 to 9. The latter were children who appeared to have special talent.

As has been noted by several investigators,¹ forming number groups, Subtest 6, is an easier process than naming aggregates, Subtest 8. The latter ability is almost at zero for the youngest subjects and advances gradually to a point slightly above 4 for oldest subjects. There is less differentiation in attainment between higher and lower intelligence groups than in Subtest 4, matching aggregates.

Though factors of group recognition measured by the several subtests develop at different rates, the composite curve for the general function, as shown in Figure 16, indicates that growth is continuous for the three successive ages. Growth for higher is greater than that for corresponding lower intelligence groups, except for three year olds in Subtest 7. Ability in sound discrimination and reproduction is so low for all groups that the exception is not significant.

Analysis of Individual Achievement.- Responses to the five tests in group recognition disclose wide variation in methods employed by subjects. Good "counters" use mature methods, while those who have not yet grasped the principle of one-to-one correspondence use immature methods. Gradations of differences in methods are not readily perceived, for identity of response is not a reliable index to identity of procedure. For example, in matching aggregates subjects may arrive at correct responses by any one of the following procedures:

- (a) responding to spatial arrangement rather than to the aggregate of unities presented;
- (b) counting by ones to ascertain the unities in the aggregate;
- (c) recognizing a smaller aggregate within the larger but, unable to proceed from these data, reverting to counting by ones;
- (d) "seeing" a small aggregate and counting on from it to determine the unities in the entire group;

¹Hermann Beckmann, op. cit., pp. 2-72; B. R. Buckingham and J. MacLatchy, op. cit., pp. 500-502. These investigators agree that forming is less difficult than naming number groups.

- (e) combining familiar "doubles" and counting forward from the point attained, e.g., two twos, two threes, or, more rarely, two fours or two fives;
- (f) "seeing" large aggregate as composed of several smaller groups, and, by combining two of these and counting remaining ones, ascertain number of unities in the entire aggregate;
- (g) stating promptly the correct cardinal number for the aggregate presented.

The order of development is from perception of simple spatial forms of aggregates toward analysis of the latter, first by counting single unities; later, by partial grouping and counting on single unities; still later, by combining partial groups; and finally, a mature stage is reached, in the prompt recognition and designation of aggregates as cardinal numbers. The maturity of method may vary from test to test with the same child, but in general maturer methods are closely associated with superior achievement.

(a) Methods of Three Year Old Children.- The aggregates 2 and 3 were matched by all subjects in the group except Subject 104 who failed to sense the problem and confined his interests to the joy of manipulating materials. Subjects 115 and 124 matched 2 and 3 but responded to other numbers by enclosing areas within which they laid handfuls of objects. Subject 117 responded to numbers beyond 4 by putting down a handful of the material indicating that the number was "many" and hence too numerous to count. This subject made the best record for the three-year group in matching grouped sounds (Test 7) and in discriminating between groups (Test 5). In two other tests he attempted "to count them" but never succeeded in counting beyond 4. These four subjects are in the lowest quartile for the group in intelligence. The methods used correspond to the first two in the preceding list of methods.

A few subjects, 102, 119, and 121, counted the unities in the aggregate presented and deliberately and slowly matched these by laying down objects consecutively until correct amounts were reached for 3, 4, and 5. While so doing they focused attention alternately between the test group and the one in process of construction. Establishing correspondence between concrete items is a difficult task for three year olds. While the procedure used advances to the second stage in the foregoing list of methods, it nevertheless shows unawareness of the group idea. The "how many" is still the counting of separate unities, not the summation of these into a group or aggregate.

More advanced methods were employed by 116 who made the highest score for the group and who ranked fifth from the highest in intelligence. He matched 2, 4, 5, 7, and 8 correctly twice and

3 and 6 once. Though counting was his usual method and one employed with more facility than any other subject in the group, he occasionally recognized a small aggregate as a whole. Two was always recognized without counting and four, as two twos. With gleeful satisfaction he pointed out six as, "That's a chee, and that's a chee!" He recognized five in Test 4 as "four with one in the middle." On the other hand, in the test for recognition of aggregates in timed exposure, Test 8, he identified only three numbers, 2, 3, and 4. Three was seen as two and one. He correctly formed 2, 3, 4, and 5 but for the latter two numbers he placed two down in front of him, and audibly counted successive objects. His methods resemble the second, fourth, and fifth (in the one last instance) methods described above.

(b) Methods of Four Year Children.- In the fourth year certain changes in attitudes and methods are apparent. The idea of "many" is designated by a favorite number, such as ten-teen, or twenty-ten, or eight-nine, or the like. Two and three are usually recognized without counting. Larger numbers are attacked at once by counting or broken up into twos, threes, or in rare instances into fours and fives. Few three year subjects see that counting offers a solution to the problem of matching, forming, or naming; few four year subjects fail to sense this fact. One subject who easily recognized two and three explained, "I know two 'cause it is like stockings, shoes, and gloves. I know three 'cause it is two shoes and one more." It is possible that she gives herein insight into methods used by four year subjects in recognizing simplest groups, that is, aggregates are recognized by association with analogous patterns rather than by grasp of twoness or threeness.

Subject 60, who scored highest for his group and ranked fifth in intelligence, readily matched 3, 4, and 5 by laying down two and adding on the remainder one at a time. In matching 6, 7, 8, and 9 he counted by twos the aggregate presented and laying down a similar number of objects he counted by ones to make sure they agreed. Before initiating this latter procedure, he repeatedly announced, "I think I'll count yours first, and then, count mine." If the two aggregates did not agree he calmly concluded, "Now I'll have to count again." In doing so he usually grouped his own aggregate into twos as he counted. For numbers above 4 in Test 8 he employed grouping; 8 he saw as "5 in one bunch and 3 in another"; 7, as 4 and 3; and 6, as 3 and 3. In each instance he counted by ones from the first named number of the two making up the aggregate to determine the total amount. For example, after recognizing 8 as 5 and 3, he counted on from 5 saying, "6, 7, 8."

Subject 56 ranked second in total test score and twelfth

in general intelligence. She not only recognized 2, 3, and 4 as aggregates in Tests 4, 6, and 8, but used this perceptual skill in the latter test by seeing 5 as 3 and 2, 7 as 4 and 3, and 6 as 3 and 3. She promptly counted forward from the first number to find the total aggregate. She saw 8 as 5 and 3, and 9 as 5 and 4; but when asked "How many?" she persisted in saying, "Can't tell." In matching sounds she was unable to go beyond 4.

Subject 51 ranked third in total score and first in general intelligence. He recognized 2, 3, 4, and 5 and used them in dealing with larger numbers. He worked with ease and without show of fatigue apparently enjoying the tasks. He made a perfect score in Test 4, and failed on but one item each in Tests 5 and 6. In matching sounds he was unable to go beyond 5 and in apprehension of group number in timed exposure he could not name 8 and 9. In the latter test he gave his answer always in the same form: "I take it to be ----." He appeared to combine small aggregates in determining the total number.

Subject 82, who had the lowest total score, ranked below the median in general intelligence. She has very little ability in counting but was able to discriminate correctly six out of seven times a larger from a slightly smaller aggregate. She matched the number 3 in Test 4 and the number 2 in Test 7. She formed 3, 5, and 6 in Test 6 after long delay spent in counting and recounting. Test 8 was beyond her.

Subject 85 could not count beyond 7 and found it difficult to concentrate upon the problems of the tests. He had, nevertheless, a keen ear for grouped sounds; in Test 7 he matched 2, 3, 4, and 7 correctly. In Tests 4, 6, and 8 he recognized 3 and 5 by counting them over and over again as if uncertain of the correctness of his response. In Tests 4 and 6 he counted the objects before him on the table and in Test 8 he nodded his head as he counted audibly with a fixed stare at the opposite wall, but correctly identified only two numbers, 3 and 5.

(c) Methods of Five Year Children.— The aggregates 2 and 3 are recognized at a glance by practically all subjects in the five year group. The number 4 is often known instantly in the same manner, though many state it as "That's two twos," or "That's three and one more." One subject labeled it always as "Two up and two down," probably from a fixed pattern learned from dominoes.

It is difficult to determine whether facile recognition of small aggregates is the result of rapid counting, combining of favorite numbers, or associating the form of the aggregate with an analogous pattern readily recalled. Subject 34, who recognized 3 in seven out of ten times, said, "That is easy. See it is one in the middle and one on each side." Subject 8 while matching 3

remarked, "That's a baby thing to do! Three means you have two more after one to make three." Again in matching 5, he said, "That's like my fingers."

A study of Table XVIII shows that in general high scores in counting accompany high scores in group recognition. Subject 14, one of the best counters in the group had worked out her own method of identifying larger aggregates. In matching 6 she said, "Here are two, and here are two, and that's two twos. Here's one more, that's 5; and one more. That's 6 now." She was one of the few who correctly apprehended 8 and 9 in Subtest 8. She saw 8 as 4 and 4 but counted forward by ones from the first 4 to reach 8; she recognized 9 as 4 and 5 but counted in the same manner as for 6, described above; that is, two and two more, and counting forward by ones until 9 was reached. When that point was reached, she heaved a sigh and said, "That sure was hard."

Subject 4, who made the highest total score in the group of forty five-year subjects had a perfect score for Tests 4, 5, and 6; he failed on one response in Test 8. In the latter test he recognized instantly 4, 5, 6, and 8 but saw 9 as 5 and 4 and counted the entire number from one. Several notations indicate the complexity of his mental processes. When forming 8, in Subtest 6, he explained, "I can make 8 in lots of ways, but it is easiest to make it, one 4 and another 4." He demonstrated that if 2 be taken away from one four and put with the other, eight would then be 6 and 2; or if 1 were taken away from 6 and put with 2, eight is then 5 and 3. These facts he illustrated with objects at hand.

The foregoing analysis indicates growth in more mature methods of dealing with aggregate number. Facility and ease in counting have matured into ready recognition of small aggregates of two, three, and four items; these are employed in thinking of larger numbers, that is, 4 becomes two 2's and 8 two 4's. Although progress in some instances is extremely slow and irregular and recognition of small aggregates is repeatedly displaced by primitive counting, nevertheless, five-year-old subjects are in the main, on the way toward higher levels of abstract thinking, namely, that of group recognition.

Summary: Growth in Group Recognition in 3-6 Year Age Levels.- Analysis of individual subject records for the three-to-six year comparable age groups yields a basis for the following characterization of growth in recognition of aggregates:

- (1) group recognition develops slowly and gradually and appears first among the brightest subjects of the age group;
- (2) recognition of the aggregate 2 is evidenced among few in third but among all subjects in fifth year; 3 is known to the brightest in the third, to approximately two-thirds

in the fourth, and to all in the fifth year; 4 and 5 are recognized as aggregates by one or two subjects in fourth and by a slightly greater number in fifth year;

- (3) large numbers are variously interpreted: in third year, as "many" without any attempt to determine their items; in fourth year, as "many" but designated by favorite large numbers; in fifth year, as groups to be counted by ones or broken up into smaller aggregates;
- (4) recognition of aggregates is closely associated with maturity in rational counting which is a sine qua non of its development;
- (5) recognition of group number is a more abstract and complex method of dealing with quantitative experience than is counting.

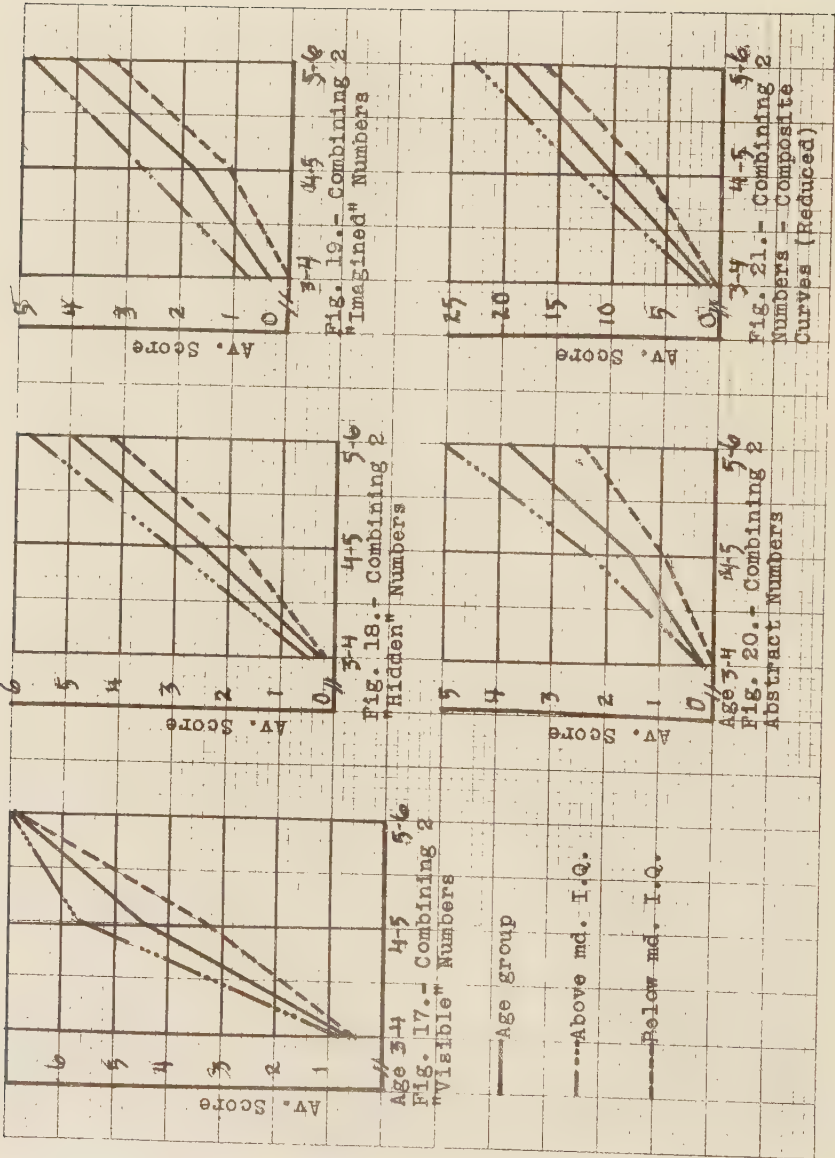
Combining Aggregates of Two and Three Numbers

Combining aggregates was a difficult process for children included in the three comparable age groups. Distribution of scores for the eight tests involving the process (Subtests 9-16) appear in Table XVII. These data were plotted in curves presented in Figures 17-26; the first five figures give growth curves for combining two aggregates and the next five those for combining three aggregates. Data for entire age-group are designated by full drawn lines, those for higher and lower intelligence groups by broken and dotted lines respectively. Scores and ages are read in the same manner as in preceding figures.

It is evident from a comparison of curves for combining two numbers (Figures 17-21) with those for combining three numbers (Figures 22-26) that attainment is greater in the former than in the latter. Analysis of characteristic methods used and recorded on test-forms offers explanation.

(a) Ability: Three-Year Level.- Few three-year subjects grasped the meaning of the task in combining two aggregates and all fail to see it with three numbers. Subject 124 was equally confused in the two situations. In the problems, 2 and 1 more, 5 and 2 more, and 4 and 2 more and 2 more, he gave the same response, namely, "3—4—10." Repetition of this response seemed to be prompted more by the pleasure of saying the jargon than from any sense that it might function in the situation. Table XIX shows that his score in counting does not exceed 3.

Subject 121, who counted to 4, knew that 2 and 1 more are "chee" but numbers beyond 4 were "lots." Subject 101 who said 2 and 1 are 3, after he counted by ones, answered all other problems in Tests 9-12 with either this same response or 10. The latter he



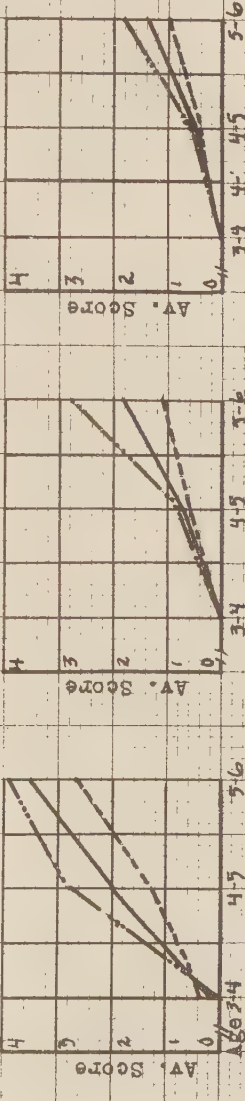


Fig. 23.- Combining 3
"Hidden" Numbers

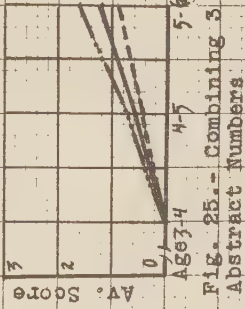
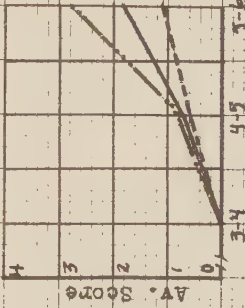
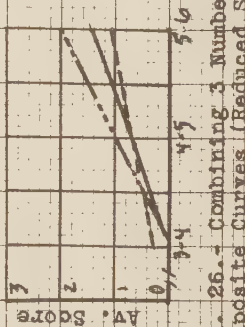


Fig. 26.- Combining 3 Numbers -
Composite Curves (Reduced Scale)



— Age group — Above md. I.Q. --- Below md. I.Q.

used for larger numbers and the former for smaller ones thus indicating recognition of difference in spatial factors involved. Subject 116 recognized groups of 2, 3, and (in some situations) 4, but was unable to employ this advantage in problems of combining. He made several futile attempts to meet the situation by counting.

Ability in combining for the entire group is very meager. The higher intelligence group made zero scores in four tests and the lower intelligence group in five of the eight tests.

(b) Ability: Four-Year Level.- In the fourth year, though seven subjects scored zero or one in Subtests 9-16, the majority of the group sensed the task and found ways of attacking it. Typical methods employed by brighter subjects are those of Subjects 51, 60, and 98.

Subject 51, a fair counter who reached 20 by rote and 13 by rational counting, counted silently over and over again before announcing his results with the expression, "I take it to be ----." He scored in each test, lowest in two tests (13 and 16) dealing with combining three numbers. In this latter process he combined the first two numbers by counting and added on the third as though it were a single unit, that is, 4 and 3 and 2 are 8, 4 and 2 and 2 are 7. This same tendency was also shown in combining two numbers, 7 and 2 are 8, 5 and 4 are 6. He used both 7 and 9 to express "many."

Subject 60 counted both by rote and rationally to 49. He recognized aggregates less than 5 and counted forward from them to ascertain the sum of two or more combined. He combined 4 and 4 thus: 4, 5, 6, 7, 8. He combined 5 and 4, as "4 and 4, 8, 9." His grasp of 4 and 4 as 8 is not always the same for occasionally he counted 4 by ones. In one problem he saw 8 and 1 as "4 and 4, 8, 9," but seemed uncertain of the correctness of his response until he counted from 1 to check his answer. In Subtest 13 he translated 4 and 3 and 2 as 5 and 4 and combined them as 4, 4, 8, 9. His high scores in combining abstract numbers is due in large measure to memorization of combinations under instruction of his nurse. He employs both primitive counting methods and the more advanced method of counting forward from small aggregates. Like Subject 51 he sometimes adds a group as though it were a single unit, as 7 and 2 are 8, 5 and 2 are 6.

Subject 98 counted by rote to 79 and to 29 rationally. She is a good visualizer. She readily recognized groups 2, 3, and 4 and used them in a limited way to aid the problem of combining. When asked how she always knew that 6 and 3 are 9, she replied, "Easy! Six has 3 and 3—and then another 3. Count them all up on the wall." Her control of group recognition, however, is not adequate enough to function in all cases of combining. Like the two

preceding subjects she combined 7 and 2 as 8, 5 and 2 as 6, and 4 and 3 as 5. Such answers may be a function of earlier counting habits. Combining for this subject is largely a counting method aided at times by recognition of small groups.

The following three subjects, 74, 72, and 73, who are triplets, used immature methods in combining aggregates. In rote counting they reached 10 or 11 and in rational 4, 8, and 12 respectively. Subject 74, though timid and frail for two years after birth, was physically up to par and rather daring and excessively inquisitive. While the examiner was filling out the test-form record he investigated cupboards, shelves, and desk drawers without molesting in the slightest their contents. His memory was good; for example, he knew in which of the several places additional test-forms were kept, which drawer held a supply of kindergarten materials, and the like. None of this information was contributed except in response to specific questions. His mother says that he makes elaborate, secretive plans without divulging his intent; he is "always full of mischief, but seldom caught red-handed." For one so clever in these ways he knows little in number. He combined 2 and 1 by counting 1, 2, 3 but failed on other problems. Observing the examiner making notations on his record and probably sensing his poor showing, he broke his usual reserve to say, "I know some numbers that you didn't asked me." Encouraged to relate these he said in parrot fashion: "2 and 2 are 4; 3 and 1 are 4; 3 and 2 are 4." These had been taught by an anxious mother who fears that he is not learning "his numbers." Five, he characterized as a "bunch with one in the middle." The number ability of this subject has not advanced much beyond that of three year olds.

His brother, Subject 73, is independent, quick tempered, super sensitive, and leader of the three children. He had very poor control of rational counting—skipped items in the objective series, and tolled out 4, 5, 6 from the subjective series as he pointed to one single item in the objective series. He indicated two by tapping opposite sides of the table and counted 4 by touching all four sides of the table slowly and deliberately. After much tapping and retapping around the table he announced that 2 and 1 are 3, and 5 and 2 are 6. The answer to the latter combination in another test he gave as 7. He is as immature as his brother in the process of combining aggregates.

Subject 72 is quite different from her two brothers. She is as dependent as her brother (Subject 73) is independent, and as artlessly frank as Subject 74 is secretive and reserved. She became confused in rational counting, adding terms at random after reaching 11. In combining she said: 4 and 4 are 5; 4 and 3 are 5. Announcement of these results followed much laborious counting

and recounting of imaginary spots out in space at which she stared fixedly while counting. The immaturity of these three subjects is closely related to inability in rational counting which does not surpass that of three year olds.

Progress in fourth year, as noted in the preceding analysis of typical individual records, may be characterized as further attempts to apply rational counting to the problem of combining aggregates and among brighter subjects, with use of small groups as aids. Though advantage contributed by the latter appears extremely slight for the group as a whole, nevertheless, it indicates the rudimentary stage that develops more clearly in the fifth year.

(c) Ability at Five Year Age Level.- The most immature five year old child was Subject 25, who counted to 16 though she appeared to have little idea what it was all about. She said 2 and 1 are 3 after a long time and then demanded that the examiner wait until she count again. In other combining problems she counted the first aggregate presented and disregarded the others. For example, 5 and 2 are 5; 6 and 3 are 6; 8 and 1 are 8; 3 and 1 and 2 are 3; and so on. If objects were visible she counted them rationally, and if hidden, she transferred the counting to her fingers or to nodding her head. In using the latter device she usually gave one fleeting glance to exposed objects and counted a mental image of them by nodding her head. The activity of bobbing her head was so absorbing that she not infrequently lost track of counting and announced random numbers as answers. Apparently she thought counting might solve the situation but she was unable to use the method effectively.

Subject 3 is at the opposite end of the group from Subject 25 both in intelligence and in number ability. She has devised a counting method that works more effectively. She counts silently by 2 to 4 or 6, and then counts by ones to the goal sought. For example, she combines 7 and 2 as follows: "2, 4, 6, 7, 8, 9"; she adds 6 and 3 as 2, 4, 6, 7, 8, 9. In adding 2, 3, or 4 to five she begins with 5 and counts by ones until the sum is reached. At times she made the error noted in preceding cases: 5 and 2 are 6; and 5 and 3 are 6; 4 and 3 are 5. This type of error, characteristic of less maturity in counting, possibly may be an attempt to combine two groups in a single act without yet being able to see that the size of the second group must be considered. She "likes" numbers and works with ease and confidence.

Subject 45 counted accurately to 39. She recognized groups of 2, 3, 4, and 5 and explained that "I count them on my fingers lots, so I know them well." Shown 5 and 4 and asked "How many" she became serious and said, "Let's see, 5 and 5 are 10 and now 5 and 4? — That's one before 10." She counted silently so rapidly

that it was difficult at times to detect the process. She gave the answers to 5 and 3 almost instantly and when asked how she did it so fast, she said, "I just say 3 and then count fast (silently) and move my head as I count." Realizing that she had an appreciative audience she shyly added, "I know lots of numbers" and then rattled off the following:

3 and 3 are 6

2 and 2 are 4

4 and 1 are 5

5 and 1 are 6

3 and 2 are 5

5 and 2 are 7

5 and 5 are 10

10 and 10 are 20

In adding 4 and 3 and 2, she interrupted herself with "Ugh, that's the same as 5 and 4; just as I told you, it is the one before 10." She added 4 and 2 and 2, as 4 and 4; and 3 and 2 and 1, as 3 and 3. The actual combining was done by counting forward by ones in each case from the first named aggregate.

The explanation of her proficiency seems due in no small measure not only to superior intelligence, but to previous instruction. In speaking of the matter, she confided that, "Aunt Flo shows me numbers." (A long descriptive account of the location of her aunt's house with reference to her own "little house back of the garden" followed.) "I wake up every morning before my mother does but I lie still, for she will scold me if I wake her up. Then, I count the pieces in the bed-quilt." The maturity behind the explanation is as amazing as the fact of the application of numbers taught by her aunt. Reflection, in the face of a problem situation and adjustment to it in the case herein described, is characteristic of much older children. Few would deny that this child has her number learning well in hand and that her maturity of methods is predictive of later outstanding success.

Summary: Process of Combining Aggregates.- Few three year olds have any understanding of the combining process. Some, however, did attempt to perform the task by employing counting methods. One year later many subjects sensed the task and used counting methods generally. A few progressed further and recognized small aggregates as bases from which the forward counting started.

The more mature among the five year olds grasped the idea that combining is augmentation from some simple familiar base. They exhibited varying degrees of skill in applying this idea as follows:

- (1) recognizing at sight, among aggregates to be combined, a known small base such as 2 or 3 and counting forward from this by ones;
- (2) counting by twos to a point of vantage from which augmentation by ones continues;

- (3) grouping small aggregates to form a favorite base and proceeding from this point by doubling or by primitive counting;
- (4) using combinations previously memorized as aids in combining two or more aggregates.

Less mature subjects continue the counting methods of three and four year olds. In its most primitive form counting proceeded with the aid of actual concrete objects,¹ or at a more advanced stage by substitution of fingers or imaginary spotting arrangements upon which attention was focused during counting. Counting by means of such tallies in some instances had become a deft skill not easily detected while in progress.

The tendency, noted also in fourth year, to add a multiple aggregate as a single unit, persists. This possibly is a recognition that the group as a unit is to be added but without, as yet, sufficient insight into a method for carrying on the process.

Of three-to-six-year old subjects, as with older subjects studied by Judd, it may be said:

"There is, however, a vast difference between being able, on the one hand, to recognize familiar groups and being able, on the other hand, to perform the elaborate mental act of explicitly defining the number in such a group."²

Interdependence of Three Processes.- The detailed analysis of the three processes, counting, recognizing aggregates, and combining aggregates, makes evident certain generalizations. Rote counting has been shown as having priority of development and essential contributory relation to growth of the other two processes. Rational counting is a complex mental process dependent upon grasping the idea of correspondence, constant between its two most important aspects—number names and discriminated objects. Rote counting involves the memorization and accurate recall of a fixed order of numeral terms; and rational counting requires application of these terms in a one-to-one relation to items of an objective series.

Characteristic errors in the early learning stages of counting reveal such difficulties as failure to learn and correctly recall the terms of the subjective series, confusion in matching

¹Francis Galton, Narrative of Explorer in Tropical South Africa, p. 133. London: John Murray, 1853. This primitive form of counting Galton found in bartering with a Damares native who refused to continue the transaction unless permitted to match sheep with sticks of tobacco for which they were being traded.

²C. H. Judd, op. cit., p. 99.

the two series correctly according to the principle of correspondence, overlearning terms of subjective series in such a rhythmical fashion as to interfere with correct correspondence and other like errors noted in the foregoing analysis.

Recognition of aggregates, though greatly facilitated by superior ability in rational counting and developed from rational counting in its initial stages, involves more complex mental processes. Group recognition with youngest subjects does not, except in a few cases, extend beyond two, but with the older and brighter subjects recognition extends to three, four, and in some few cases to five. The latter subjects by numerous methods and insights have come to regard the group as having a distinct entity of its own dependent upon but not the same as that of its several constituent items. This is a new relational factor as significant for later development as the principle of correspondence is in counting. Methods that tend to make this new relational factor articulate employ matching of small groups in pairs or doubles; recognition of aggregates by general form, symmetry, or familiar analogies; extension of the pairing function into rational counting by twos or larger units; a breaking up of larger groups into smaller ones readily counted or "seen"; these and other insights aid the singling out of a new factor that in turn becomes an essential subsidiary factor in the growth of ability to combine aggregates.

The process of combining among three to six year subjects is in a very immature stage. For the youngest subjects little advance has been made and for them, as for the four year subjects, combining is largely a process of counting. Individual competency in the fifth and what there is below this level in third and fourth, is distinctly greater with two than with three numbers. This may be explained in part by the greater complexity in the mental process involved in combining three numbers since in this case the partial sum resulting from combining two numbers must be held in mind until the third number is added to it and the total sum is obtained. Ability in the combining function differs with different types of experience. Combining visible objects is much less difficult as a process than combining the same number of objects seen in timed exposure, or imagined, or as abstract numbers. The obvious explanation offers that in the first situation the ready application of the familiar and primitive method of rational counting is not inhibited.

The foregoing analysis makes clear the conclusion that growth in number ability at these immature ages within the limits of the investigation is very slow and gradual and dependent upon

many factors. Brightness is one such factor that strongly influences superior ability at each age-level. Furthermore progress is intimately interrelated and interdependent upon mastery and insight into more and more mature methods of dealing with number experiences. These maturer methods are evolved with individual understanding of relational factors existing between numbers.

CHAPTER VI

SUMMARY AND GENERAL CONCLUSIONS

The foregoing investigation concerned itself with the study of development of number ability in pre-school children with respect to processes of counting, recognizing of aggregates and combining aggregates. The study involved a sequence of problems including: (1) organization, administration, and validation of a test-battery; (2) analysis of obtained results to determine correlated factors; (3) study of growth in relation to the factor of general intelligence; (4) analysis of age-group and individual development in each of the three processes; and (5) re-evaluation of the literature dealing with the development of these processes in the race and in the individual.

Organization, Administration, and Validation of the Test-Battery

A set of tests in counting, discriminating number pictures, and matching geometrical forms was first devised for purposes of re-teaching an aphasic who had lost control of number ideas and processes. The tests were tried out later on a group of thirty-five children ranging in age from twenty-seven to eighty-one months. Still later, the tests were modified by eliminating several original tests and substituting more adequate ones based on those designed by other investigators. This stage of the study resulted in the number-test described in Chapter II.

The test was administered first to one hundred children between the ages of forty-eight and seventy-two months and later to twenty-five others between the ages of thirty-six and forty-eight months. Subjects were so selected as to yield groups of twenty-five for each six-month period within the total age range. All subjects were enrolled in kindergartens or nursery schools of southern California. Two sets of scores, Score A and Score B, were obtained—one from the first administration of the test and the other from a re-test. All subjects were given individual intelligence tests.

Analysis by correlation methods yielded data (1) relative

to validity and reliability of the number test, and (2) relative to significant correlation between general intelligence and attainment in the test.

Correlation between the two sets of scores gave a coefficient of reliability of $.95 \pm .01$ for age-groups combined and $.95 \pm .01$ for age-groups taken separately. Range of talent did not influence the magnitude of the coefficient of reliability. Correlation data are shown in Table V and represented graphically in Figure 1, Chapter II. Additional evidence of reliability is offered by the index of reliability and by the values for standard and probable errors of measurement. These three indices justify the conclusion that the obtained scores in all probability do not deviate from corresponding true scores in significant amounts and that the number test is a reliable means of measuring the three processes.

Validity was sustained by correlations with the following criteria: (1) M.A. and I.Q. scores on the Stanford-Binet and the Detroit Kindergarten tests, considered separately and together; and (2) chronological age. The coefficients range from $.35 \pm .05$ with C.A. as criterion, to $.71 \pm .04$ with M.A. (Stanford-Binet) as criterion. Correlations with teachers' ratings gave a coefficient of $.65 \pm .03$. These coefficients, by the general rule, are positive and significant and establish a fair degree of validity for the test-battery herein employed. The low correlation with C.A. and the relatively high correlation with M.A. and I.Q. indicate that development of ability was more closely related to increase in intelligence than to chronological age. The study of the two foregoing problems appears in Chapter II.

Analysis of Development in Successive Age-Groups of Comparable Intelligence

The third problem dealt with the analysis of growth in number ability in relation to the factor of general intelligence. In order to secure a measure of the three number processes at more immature stages than those found at four and five year levels the same tests--number and intelligence--were administered to twenty-five three year old nursery school subjects. For purposes of comparison, the intelligence factor in all three age-groups was equated. By this process age-groups were made comparable, with a median I.Q. of 110.

Year by year growth was first analyzed for age-groups as wholes and next, for those above and below the common median I.Q. Growth in functions studied was shown by the means and medians to be continuous and distinctly differentiated at successive age-

levels. Rate, as measured by yearly increment in median performance, was uniform while that of mean performance increased slightly for the oldest group. Attainment for age-groups of higher intelligence reached higher levels throughout the three successive years than that for corresponding groups of lower intelligence. Furthermore, growth curves for these groups maintained a fairly constant relation to each other, indicating the relative constancy of the intelligence factor.

Variability within age-groups increased from year to year but the increment was not constant owing to lack of difficulty range in several test items.

Acceleration in growth appears at fifth and in less degree at fourth year levels but is greater in amount for higher than for lower intelligence groups. The spurt in development, contrary to finding of other investigators, was shown to be at fifth rather than at fourth year level and to be correlated with proficiency in rational counting process rather than with recognition of the group number three as reported by Stern.¹ Verification of these findings is offered in the analysis of development of the three processes that follows herein.

The technique of selecting comparable age-groups, the statistical treatment of obtained measures to determine sampling reliability, and the interpretation of findings are presented in Chapter III.

Analysis of Age-Group and Individual Growth in Three Processes

The fourth problem concerned itself with analysis of number ability in relation to development in three component processes: counting, recognizing aggregates, and combining aggregates. Data for this problem included individual and comparable age-group scores for the sixteen subtests.

Counting involved complex mental processes that matured very gradually during the three successive years under investigation. At three year level approximately 70 per cent of the group counted to four, and 7.6 per cent bridged the first decade. A year later 51.0 per cent bridged the first decade, 18 per cent bridged the second decade, and 12.0 per cent counted into the third and fourth decades. In the five-year group 22.5 per cent reached numbers between thirty and sixty and 12.5 per cent counted to one hundred.

¹William Stern, op. cit., p. 416.

Counting an objective series was more difficult than rote counting. The former involved the idea of correspondence between the item of two series--the objective and the subjective. All subjects steadily advanced from three to six years in ability to match the items of the two series. Until the third or fourth decade is reached, achievement in rational is slightly less than that in rote counting. The smallness of the difference is outstanding and at variance with earlier investigation though in agreement with more recent studies.

Counting backward does not appear as a measurable factor until the fourth year and then only in the upper intelligence group. A year later, approximately one third of the group had accomplished the task from 5 and one tenth from 20. It should be noted that the latter is one of the tasks for eight-year-olds in the Stanford-Binet test.

Analysis of individual growth revealed that outstanding difficulties in counting included instability of control of the subjective series and lack of mastery of the idea of correspondence.

Analysis of growth in recognizing aggregates showed that this process is closely associated and dependent upon maturity in rational counting. Ability appeared first among brightest children. Recognizing the aggregate 2, evidenced by few third, is known to all fifth year children; 3 is known to the brightest in the third, to approximately two thirds in the fourth, and to all children in the fifth year; 4 and 5 are recognized as aggregates by one or two children in fourth and by a slightly greater number in the fifth year.

Few three-year-old subjects had any understanding of the combining process; four-year-olds sensed the task and used primitive counting methods in performing it; five-year-subject grasped the idea that combining is augmentation from a familiar base. Less mature subjects at each age-level continued primitive counting methods.

Throughout the analysis of the three processes there was a marked tendency for children of higher intelligence to achieve better averages, to encounter fewer difficulties and to have, in general, greater facility in dealing with number. Though marked as this tendency is it must be noted that it is not so infallibly constant as to warrant the conclusion that superior number ability always accompanies superior intelligence.

Data for the following summary appear in Chapter V of the dissertation but not in the essential portion thereof.

Genesis of Number Development in the Race and in
the Individual: Survey of the Literature

The fifth and last problem of the present investigation considered the development of number in the race and in the individual as found in the literature of the subject. The sources of data are meager and scattered but a consideration of that found available gives basis for the following conclusions based on the study that appears in Chapter V.

Earliest beginnings of number lie far back in antiquity and the theory of their conception is as yet full of riddles. Progress, nevertheless, can be traced through records of extinct tribes and study of extant peoples in primitive stages of development. From a consideration of such sources stages of advancement from lowest barbarism to early civilization may be seriated in the following summary:

- (1) discrimination of unity from plurality in concrete collections;
- (2) perception of parity, duality or equivalence of objects associated naturally as pairs; perception is less concerned with "twoness" and more with similarity and resemblance;
- (3) recognition of a few units in a homogeneous plurality by discrimination of one or more separate unities apart from the collection, that is, noting one, another one, another one, many;
- (4) numeration in one-to-one correspondence by means of tallies or non-verbal aids, e.g., matching objects, one at a time, with counters--pebbles, knots in a cord (Peruvian equippa), notches in a stick, digital counters, and the like;
- (5) formation of short count scales by augmentation from simple radices, e.g., one, two, two-and-one-more, two-two;
- (6) substitution of descriptive terms (later of mathematical symbols) for non-verbal tallies;
- (7) discrimination of group or cardinal number by process of counting;
- (8) combining of simple units and small aggregates by the process of augmentation.

Progress in stages 1, 2, and 3 is on the basis of perceived concrete realities. Discrimination is concerned with singling out material objects as "this" and "that" from the "many" or plurality. Abstraction is practically at zero for this is a perceived and not a conceived unity. The primitive mind perceives the discontinuous in anthesis to the continuous--such is the primitive equivalent of unity and plurality.

The next three stages, 4, 5, and 6, mark the emergence of the idea of correspondence and the development of count scales. Limited development for some tribes was confined to the binary scale; others, by augmentation to the primary radix, 2, arrived at a quinary scale; more advanced peoples using radices of 5, 10, or 20 reached greater independence in serial counting. At first, extension of number ideas to higher points could operate, only, when sustained by concrete material accompaniment; later, tallies, verbal terms and finally symbols gradually and by slow degrees were substituted for concrete objects. The simplicity and solidarity of these count scales were maintained by the rigorously determined unity from which any term in the scale took its departure and back to which it could be readily referred. Thus in the quinary, decimal, vigesimal, or even sexagesimal scale the radix bears a constant relation to unity and in turn all higher terms in the particular scale are readily visualized in relation to the radix and to unity. In the decimal scale, for example, fifty or one hundred bear a constant relation to unity and to the radix ten. Thus count scales gave a tool by means of which progress to higher levels of abstraction was made possible.

Stage 7, discrimination and recognition of group number as a development of the counting process and finally stage 8, rudimentary addition of simple aggregates advanced number thinking to more abstract levels. Once the idea and method of succession in serial order were grasped the way was open for the development of the processes of augmentation (addition) and diminuation (subtraction). Thus correspondence and succession, the two principles basic to all mathematics, were evolved from the hierarchy of mental processes developed among primitive peoples in dealing with quantitative experiences.

The child, as did the race, arrives at a sense of number by slow and very gradual degrees. Three stages of progress are noted by Stern, (a) the idea of individuality or recognition of single familiar objects; (b) plural ideation or the discovery of likeness between members of a class; and (c) collective, group or cardinal idea of number. The emergence of ability to recognize individual objects he believes is linked with the "relating activity" that "can distinguish points of similarity in several impressions to the exclusion of points of non-agreement." His tests for this function indicates its appearance in subjects of thirty-three months. Plural ideation he applies to that stage marked by the discovery and recognition of likeness among objects of a class.

This later stage Phillips designates as the "embryonic" counting stage, the motor response to a consciousness of succession

which has resulted from the rhythms and recurrences of life. The latter, he asserts, not only give the fact of succession but also the idea of succession.

In this characterization both Stern and Phillips ignore the influence of social factors upon numerical development. This latter point of view is stressed by Judd who, though readily conceding that first discriminations, with the child as with the race, result from maturing personal interests, shows that number is not a product of instinctive tendencies but the result of discriminative guidance by social rather than personal considerations.

Royce states that counting is not merely attaching mechanical names to objects, nor "the mere collection of acts of synthesis by which we each time add one more, in mind, to the collection of objects so far counted." Counting and keeping tally is possible only as the child is able to keep constant the correspondence between the objects discriminated and the ordered series of counters or number names. Judd, Stern, and Royce emphasize the fact that rational ideas of number assume their abstract quality only when the relation is recognized as no longer dependent upon concrete objects between which it exists. Recognizing the relation between corresponding items in the number-name series and the discriminated objective series is a relational form of consciousness that holds "two factors together in a certain form of organized experience." Royce analyzes this mental phenomenon as a process of one-to-one relation or correspondence, a principle fundamental to all mathematics. Brownell, on the basis of experimental data, characterizes this phase as rational counting which includes a series of mental activities such as knowing correct order of number names, maintaining one's place among objects counted, and attaching number-names to these in a one-to-one correspondence.

The significance of general intelligence in number development has been noted by such investigators as Binet, Hartmann, and others. As early as 1880 Binet recognized the relation between facility in counting and maturing intelligence.

Eckhardt examined school entrants and concluded from his findings that the stage of development of number facts and skills of school entrants varies greatly and is dependent in large measure upon environmental opportunities. In this view Meumann concurs and says variation may be traced to preschool influences that lead to a knowledge of number relations and stimulate the development of number concepts.

The recognition of cardinal number involves a further and higher stage of abstraction than is the case in rational counting. Stern observes that it takes much experience in counting on the

part of the young child before he comes to realize that the last-named number is also the general summarized term for the group of objects. Beckmann, Filbig, Descoeudres and more recently, Buckingham and MacLatchy, Woody and others have analyzed separate factors involved in recognition of aggregates and report in detail analysis of gradations of method of apprehension from simple and primitive counting to mature spontaneous recognition of cardinal number value. These studies for the most part dealt with school entrants or with subjects prior to school instruction in number. Brownell studied methods employed by older subjects in grades two to six.

Once the idea of aggregates is firmly grasped, relational values between groups are gradually perceived. As ability in generalization develops, progress to higher levels continues and groups are combined, at first by the process of simple augmentation to a small familiar base. Augmentation proceeds at first with single units and later with larger amounts.

In the analysis of growth of number ability presented in Chapter V, it has been shown, that with the child and with the race, progress, beyond mere rudimentary beginnings, is slow and very gradual and at all stages dependent upon a hierarchy of generalizations and abstractions.

General Conclusions

An analytical study of the year-by-year development of ability in the three processes—counting, recognition of aggregates and combining aggregates—gives reasonable basis for the following conclusions:

- (1) the counting process appears earlier than that of group recognition, or combining aggregates;
- (2) recognition of aggregates is functionally related to facility in rational counting;
- (3) each of the three processes involves a succession of inter-related mental activities the maturation of which is accelerated by general intelligence;
- (4) ability to grasp abstract relations of correspondence appears among some three year and among all five year subjects;
- (5) growth shows spurt in development coincident with grasp of relational factor of correspondence;
- (6) ability is more closely correlated with general intelligence than with chronological age;
- (7) use of the principle of augmentation does not appear before fourth year and then only among those of higher intelligence;

- (8) superior ability is found generally, though not always, among subjects of superior intelligence;
- (9) numerous non-brightness factors, such as health, general emotional attitudes, social environment and training or guidance in learning influence development of ability;
- (10) successive stages of development of ability in the child resemble, in general, corresponding stages of development in the race.

While the foregoing are not to be regarded as final conclusions without further verification, the investigation indicates that, with subjects herein described, the establishment of initial and progressive abstractions and generalizations is basic to development of numerical ability.

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APPENDIX

NUMBER TEST FOR PRE-SCHOOL AGES

SERIES A--COUNTING

Subtest 1--Rote Counting

Have subject count by ones. Examiner says, "Let me hear how far you can count. Begin counting in this way, 1, 2, 3 and go as far as you can." Do not count beyond 3 for the subject. Record the score, which is the last number in the series successfully counted without intervening error, on the test-form. Note the following: (1) was the task understood; (2) did other numerals displace the correct series; (3) was counting done unhesitatingly or haltingly? Repeat the test after an interval.

Subtest 2--Rational Counting

Use plain cardboard discs, small cubes, or the like and arrange twenty or more in parallel rows before the subject. If the subject scored high in Test 1 use a number proportionately greater to insure testing limits of ability. Say, "Count these ---- in this way, 1, 2, 3, and so on. Touch each one as you count it." The score is the last number successfully counted. Record score and note: (1) deviations from correct serial order; (2) synchronization of numerals with corresponding objects; (3) tendency to group counts into rhythm. Repeat test after an interval.

Subtest 3--Counting Backward

Have the subject count backward from 5, 10 and 20. Say, "Let me hear you count backward like this, 3, 2, 1. Begin with 5 and count back to 1." If response warrants, continue in like manner with 10, and 20. Record score as 0, 5, 10, or 20 according to success.

SERIES B--GROUP RECOGNITION

Subtest 4--Matching Number Groups

Use splints, discs, or other counters against a contrasting background. Give the subject 20 or more and keep a like quantity. Ask him to turn his back until the "game" is ready. Arrange 3 counters in a horizontal row on the table saying, "Turn around now. You place as many ---- on your part of the table as I have on mine." Have him place his counters on part of table diagonally opposite that used by examiner. Repeat same procedure with the following numbers: 6, 4, 9, 5, 8, 7. Before successive presentations say, "Turn around again while I change my picture," and, "Now turn back and make your picture match mine." Score 1 for each correct response and observe whether subject: (1) removes all counters and builds number by ones; (2) counts number presented and his own to ascertain likeness; (3) builds up or subtracts from his former number to make new number; (4) groups counters by twos or threes in building numbers.

Subtest 5--Discriminating Between Two Aggregates

Have three cards (3 x 5 inches) for each number from 2 to

9. Represent the numbers in pattern arrangement by circular spots ($\frac{3}{8}$ inch). Putting down a card containing 4 spots say, "This card is for you," and placing a 3 spot card in front of examiner, continue, "and this one is for me." Continue to expose in brief succession the other three and four spot cards repeating the same question: "Is this like your card or is it like mine?" Present numbers in the following succession and note that any two aggregates differ in size by one only. Record score of 1 for each correct response and observe methods used.

Subtest 6—Forming Number Groups

Place 20 counters (pennies, splints, beads, etc.) before the subject and say, "Give me 5." If subject succeeds ask for numbers above 5; if he fails, continue with those below 5. Record response and the method of performance. Use following sequences: 5, 8, 6, 9, 7 or 5, 3, 1, 4, 2.

Subtest 7—Reproducing Sound-Groups

Use two small tap-bells and as these are shown say, "Here is a bell for you and one for me. This is the way mine rings." (Tap it twice). "You ring yours that way." After a brief period of playing to get acquainted with the bells say, "We will pretend that these are telephones. I'll telephone to you from my house and you ring just the same number of times to let me know that you have heard." Conceal the bell under the table and tap 3 rings. Record the response and continue with 2, 5, 1, 4, 6, 8, 7, 9. Repeat the test after an interval and note significant methods in the performance.

Test 8—Naming Aggregates in Timed Exposure

Arrange numbers from 3 to 9 in spot pattern on cards (3 x 5 inches) in such manner as to favor regularity of form, i.e., domino for 5, triangle for 3, quadratic pattern for 4, etc. Use time exposure of two seconds for all numbers in following order: 5, 4, 7, 9, 8, 6, 2. Record oral responses and after each ask: "How did you know it was 7? or 5?" Note how aggregates were perceived—as wholes; as small groups to which were counted—on other units to make whole; as a succession of ones. Did subject appear to count memory image following exposure of card containing number? Repeat the test after an interval.

SERIES C-1

Subtest 9—Combining Two Visible Aggregates

In the eight tests that follow pattern arrangement is avoided. Use counters (pennies, beads, splints, etc.) to represent the combination, 2 and 1. Say, "Look at these ---- and tell me how many are here." Note method of combining—Is combining done by counting both aggregates by ones; or does counting proceed from one aggregate as a base, or is the sum "seen" instantly? Continue with following combinations, recording responses and observations: 5 and 2; 4 and 4; 6 and 3; 8 and 1; 7 and 2; 5 and 3; 6 and 2; 4 and 3; 5 and 4.

Subtest 10—Combining Two "Hidden" Aggregates

Use the same counters and combinations employed in Test 9. Have the subject turn his back as the groups are arranged. Say, "I have hidden some ---- under each hand." Expose those under each hand in succession for two seconds, saying: "Look at these." Then, "Now, look at these. Tell me how many are hiding under both

hands." Record responses, note methods as above.

Subtest 11—Combining Two Imagined Aggregates

Use no concrete counters. Have subject hold out both hands, palms up, and go through the motions of putting the pennies in his hands—one group in each hand. Make sure he is visualizing the situation. Record and note peculiarities of methods.

Subtest 12—Combining Two Abstract Numbers

Say, "We will not use any pennies this time at all. You tell me right off your answer. Two and one more? Five and two more? etc." Record responses as given. Observe characteristics of method. Repeat all four tests after an interval.

SERIES C-2—COMBINING THREE AGGREGATES

Subtest 13—Combining Three Visible Aggregates

Proceed as in corresponding exercise in Test 9. Enter a record for each item in the exercise. Note characteristics of method.

Subtest 14—Combining Three Hidden Aggregates

Proceed as in corresponding exercise in Test 10. Give subjects opportunity to try each item. Record responses and note methods.

Subtest 15—Combining Three Imagined Object Groups

Follow the procedure of Test 11. Place one group in one palm and two groups in different parts of the other. Record responses and note special methods.

Subtest 16—Combining Three Abstract Number Groups

Name slowly and distinctly the number groups to be combined, 5 and 1 and 2 make how many altogether? 4 and 2 and 1 more make how many altogether? Record responses and observations. Repeat all four tests after an interval.

Test-Form: Number Test for Pre-School Children

Name _____ Date of _____ School _____ Date of _____
Birth _____ Test _____

Series A—Counting: Rote, Rational and Counting Backward

Subtest 1—Rote Counting Observations: _____
(a)-1st T _____
(b)-2nd T _____

Subtest 2—Rational Counting Observations: (1) _____
(a)-1st T _____ (2) _____
(b)-2nd T _____ (3) _____

Subtest 3—Counting Backwards
From: 5---10---20 Observations: _____
(a)-1st T _____
(b)-2nd T _____

Series B—Group Recognition

Subtest 4—Matching Number Groups Observations:
A-Groups 5--10: B-Groups 2--5: (1) _____
5 - 7 - 9 - 6 - 8 5 - 2 - 4 - 3 (2) _____
1st T _____ 1st T _____ (3) _____
2nd T _____ 2nd T _____ (4) _____

Subtest 5—Discriminating between Groups
3-4; 4-5; 7-8; 5-6; 8-9. Observations: _____
1st T _____
2nd T _____

Subtest 6—Forming Number Groups
A-Groups 5--10: B-Groups 2--5: Observations:
5 - 8 - 6 - 9 - 7 5 - 3 - 2 - 4 _____
1st T _____ 1st T _____
2nd T _____ 2nd T _____

Subtest 7—Reproducing Sound-Groups
A-Groups 5--10: B-Groups 2--5: Observations:
5 - 8 - 6 - 9 - 7 5 - 3 - 2 - 4 _____
1st T _____ 1st T _____
2nd T _____ 2nd T _____

Subtest 8—Naming Groups in Timed-Exposure
Groups 5 - 3 - 7 - 4 - 6 - 8 - 2 - 9 Observations: _____
1st T _____
2nd T _____

Series C₁ and C₂—Combining Aggregates

Series C₁—Combining Two Aggregates:

		Subtest 9		Subtest 10		Subtest 11		Subtest 12	
		1st T	2nd T	1st T	2nd T	1st T	2nd T	1st T	2nd T
2	and 1	_____	_____	_____	_____	_____	_____	_____	_____
5	" 2	_____	_____	_____	_____	_____	_____	_____	_____
4	" 4	_____	_____	_____	_____	_____	_____	_____	_____
6	" 3	_____	_____	_____	_____	_____	_____	_____	_____
8	" 1	_____	_____	_____	_____	_____	_____	_____	_____
7	" 2	_____	_____	_____	_____	_____	_____	_____	_____
5	" 3	_____	_____	_____	_____	_____	_____	_____	_____
6	" 2	_____	_____	_____	_____	_____	_____	_____	_____
4	" 3	_____	_____	_____	_____	_____	_____	_____	_____
5	" 4	_____	_____	_____	_____	_____	_____	_____	_____

Observations: _____

Series C₂—Combining Three Aggregates:

		Subtest 13		Subtest 14		Subtest 15		Subtest 16	
		1st T	2nd T	1st T	2nd T	1st T	2nd T	1st T	2nd T
5-1-2		_____	_____	_____	_____	_____	_____	_____	_____
4-2-1		_____	_____	_____	_____	_____	_____	_____	_____
1-2-3		_____	_____	_____	_____	_____	_____	_____	_____
4-3-2		_____	_____	_____	_____	_____	_____	_____	_____
4-2-2		_____	_____	_____	_____	_____	_____	_____	_____

Observations: _____

Teachers' Supplementary Reports
(125 Subjects)

(The following data are included: number interest rated on five point scale; temperament types classified as A - active, X - expressive, N - non-expressive, S - stolid; family membership indicated by E - eldest child, M - neither eldest or youngest, O - only child; economic condition above average denoted by *, below #, average unmarked. Subjects are designated as in Tables II to IV, Chapter I: five-year subjects are numbered from 1-50; four-year, 51-100; and three-year, 101-125. The Schools reporting are listed on p. 157.)

Sub- ject	Sex	School	Date Rated	Number Interest	Temper- ament	Family Member	Remarks
1	G	6	5-20-30	C	N	O	Nervous; sensitive; "fibs"; "takes things"; F. motion picture director.
2	G	2	6-10-30	B	N	Y	Ill health; oldest sister, University senior.
3	G	2	6-10-30	A	AX	M	Leader, aggressive; dominates twin sister.
4	B	3	5-25-30	A	AX	E	Smart and quick; likes attention. F. Univ. Prof.
5	B	1	5-13-30	A	AX	O	
6	G	1	5-13-30	B	AX	O	
7	G	2	6-10-30	B	AX	O	Aggressive; over-disciplined. M. ex-supervisor, Kindergarten; F. Tr.
8	B	1	5-13-30	A	AX	Y	Babylsh.
9	B	1	5-13-30	C	N	O	Phlegmatic; unsocial; full of fears.
10	B	1	5-13-30	C	A	Y	Thoughtless; irresponsible; non-group conscious.
11	B	1	5-13-30	B	N	Y	Phlegmatic, lonely.
12	G	1	5-13-30	A	AX	Y	
13	G	1	5-13-30	A	AX	O	
14*	G	6	5-20-30	A	A	Y	Leader; sensitive, resents criticism; adopted; good home.
15	G	1	5-13-30	C	A	M	Retarded 2 yr., deafness from tonsils; musical.
16*	B	6	5-20-30	C	S	M	Family 7 children, 3 boys, 4 girls; F. cripple.
17	G	1	5-13-30	B	N	Y	Phlegmatic; shy; retiring.
18*	B	6	5-20-30	A	X	E	Nervous; self-conscious; nurse, Kindergarten-Tr.
19	G	1	5-13-30	B	AX	E	Sister of triplets listed below: 72, 73, 74.
20*	G	1	5-13-30	A	AX	Y	Self-reliant.
21	B	1	5-13-30	C	A	Y	Ill health; recently moved to California. Slow.
22*	G	6	5-20-30	B	AX	O	F. attorney. Grandmother spoils child.

Teachers' Supplementary Reports (Continued)

Sub- ject	Sex	School	Date Rated	Number Interest	Temper- ament	Family Member	Remarks
23	B	1	5-13-30	B	A	O	Phlegmatic; shrinks from comment, fears disapproval; frail.
24*	G	1	5-13-30	B	A	Y	Underweight.
25	G	1	5-13-30	A	AX	E	Stubborn; lacks sense humor; speech difficulties;
26	B	6	5-20-30	C	S	E	interested in nature study.
27	G	1	5-13-30	D	A	O	Nervous; frail; faulty health habits.
28	G	1	5-13-30	B	AX	M	Impulsive.
29	G	1	5-13-30	B	A	Y	Impulsive; strong likes and dislikes.
30	G	1	5-13-30	B	AX	Y	
31	B	1	5-13-30	C	A	O	
32	G	1	5-13-30	C	A	M	Frail.
33	B	2	6-10-30	B	AX	E	Attended nursery school; illness in infancy; P. mer- chant.
34	B	1	5-13-30	B	AX	Y	Very impulsive; emotional.
35	G	1	5-13-30	A	AX	E	Left handed.
36	G	2	6-10-30	A	AX	Y	Twin; F. college dean.
37	G	2	6-10-30	A	AX	Y	Twin sister of No. 36.
38*	G	2	6-10-30	A	AX	M	Initiative; keen intelligence; F. and M. separated. Nurse.
39*	B	1	5-13-30	B	AX	O	Twin; baby "patois."
40	B	1	5-13-30	B	AX	E	Twin brother of No. 40; unsocial.
41	B	1	5-13-30	B	AX	E	Frail.
42	G	1	5-13-30	B	N	E	Not group conscious. Adopted with his sister in in- fancy.
43*	B	1	5-13-30	B	A	E	Sly and secretive.
44	G	1	5-13-30	B	AX	Y	Very shy but keen minded.
45	G	1	5-13-30	B	N	O	
46	B	1	5-13-30	A	A	M	Mother ill health - Family separated. Child neg- lected.
47	G	1	5-13-30	B	AX	O	Tonsillotomy.
48	B	1	5-13-30	B	AN	Y	Tonsillotomy.
49	B	1	5-13-30	A	AX	Y	

Teachers' Supplementary Reports (Continued)

Sub- ject	Sex	School	Date Rated	Number Interest	Temper- ament	Family Member	Remarks
50	B	1	5-13-30	A	AX	Y	Wholesome; rigorously disciplined; F. mechanical engineer. Scotch. M. musical, psychol. student; F. business executive. Kindergarten atmosphere in home. Gets much attention; parents cooperative; eager to benefit child; F. importer.
51	B	3	5-25-30	A	AX	E	
52*	G	6	5-20-30	A	X	Y	
53*	B	6	5-20-30	B	A	E	
54	G	1	5-13-30	A-	AX	M	Irresponsible.
55	G	1	5-13-30	C	AX	Y	
56	G	1	5-13-30	B	AX	Y	
57	G	1	5-13-30	C	AX	Y	
58	G	1	5-13-30	B	A	Y	Aggressive; keen minded; F. and M. costume designers; Russian. Nurse. Lacks initiative; neglected M. Burdened with household duties; F., laborer, unemployed.
59	G	1	5-13-30	B	AX	O	
60	B	2	6-10-30	A	AX	E	
61	G	2	6-10-30	C	N	Y	
62	G	1	5-13-30	A	AX	Y	Cooperative but never joins in discussion; relates all at home. M. ex-chorus girl; F. much older. Initiative; very obedient; F., wealthy; M., ex-kindergartner, devoted to home and children. Lacks initiative; neglected due to M. illness; parents college graduates. Initiative; F., univ. prof. Serious "flu" illness. Enthusiastic but lacks drive; easily discouraged; dependent upon M.
63	G	1	5-13-30	C	A	O	
64*	B	6	5-20-30	B	S	E	
65*	G	2	6-10-30	C	AX	Y	
66	G	2	6-10-30	D	N	E	Aggressive. Mother married at 16, seems like sister; not interested in education. Triplet; social; underweight.
67	G	3	5-25-30	C	X	O	
68	G	1	5-13-30	B	A	Y	
69	G	2	6-10-30	B	A	Y	
70	G	2	6-10-30	B	AX	E	
71	G	2	6-10-30	C	AX	O	
72	G	1	5-13-30	C	N	M	

Teachers' Supplementary Reports (Continued)

Sub- ject	Sex	School	Date Rated	Number Interest	Temper- ament	Family Member	Remarks
73	B	1	5-13-30	C	AX	M	Triplet; sly; underweight.
74	B	1	5-13-30	C	N	M	Triplet; slightly underweight; retiring nature.
75	G	2	6-10-30	A	AX	Y	Leader, capable, aggressive; musical; memorizes readily. F. doctor; M. nurse.
76	G	2	6-10-30	C	N	Y	Ill health.
77	B	3	5-25-30	B	A	O	Bluffer; baby talk. Limiting environment.
78*	B	3	5-25-30	B	A	Y	Shy; fears people; peacemaker; too stimulated by French nurse.
79	B	3	5-25-30	B	A	Y	Responsive; twin of No. 80. Brother in first, sister in third grade.
80	B	3	5-25-30	B	A	Y	Uninterested; home unorganized; F. title writer, movies.
81	G	3	5-25-30	D	X	M	Physically well developed; hoydenish; disturber and fighter.
82	G	3	5-25-30	E	N	Y	Lacks initiative; suggestible; well liked by children.
83	B	3	5-25-30	C	A	O	Poor motor coordination; appears retarded mentally.
84	B	3	5-25-30	B	X	E	Aggressive mentally; loves attention; resents criticism.
85*	B	6	5-20-30	D	N	O	Easily overstimulated; 1½ yrs. in nursery school.
86	G	2	6-10-30	A	AX	O	F. retired.
87	B	3	5-25-30	B	X	O	Aggressive, fluent talker, initiative outstanding.
88	B	2	6-10-30	C	A	O	Adopted, M., unmarried.
89*	G	6	5-20-30	A	X	E	Best of class; takes suggestions; cries easily.
90*	G	5	5-15-30	C	A	O	Aggressive, spoiled. Parents in business.
91*	G	6	5-20-30	C	N	Y	Industrious, systematic, musical. F. composer.
92	G	3	5-25-30	C	A	Y	Nurse and grandmother in charge.
							Overshadowed by adoring nurse. F. and M. in picture industry.
							Conscientious, industrious, persistent; baby talk.
							F., physician.
							Dramatic, independent in ideas; not easily swayed.

Teachers' Supplementary Reports (Continued)

Sub- ject	Sex	School	Date Rated	Number Interest	Temper- ament	Family Member	Remarks
93	G	3	5-25-30	C	S	0	Frail; emotional; adopted orphan. Criticizes children. Tale carrier.
94	B	4	5-29-30	C	X	Y	Fond of dolls (brings all kinds, refuses to set them aside) and white mice (observes for hours). F., engineer.
95	G	3	5-25-30	B	N	Y	Youngest in class; silent; aloof; refrains from activity, fears getting "dirty."
96*	G	5	5-15-30	D	AX	0	Nervous, underweight, lovable. M. cares for appearances only.
97*	B	5	5-15-30	C	AX	Y	Pleasant, whiny and full of braggadocio by turns. Two older brothers make life a burden.
98*	G	4	5-29-30	A	X	0	Physically and mentally best in class; not live up to best; must be pushed into class activities; prefers to play nurse. F., physician.
99*	G	5	5-15-30	A	AX	E	Sophisticated; easily bored; disinterested; sunny, child-like, if interested.
100	B	4	5-29-30	A	X	0	Physically fit; mentally alert. F. in business; takes movies of ch. activities.
101	G	7	6-25-30	B	X	0	F. salesman; M. vice-principal high school.
102	G	7	6-25-30	C	X	0	Both parents are deaf-mutes.
103	G	7	6-25-30	C	A	E	F., bed-ridden; M., teacher.
104	B	7	6-25-30	C	X	0	Adopted; immature.
105*	G	5	5-15-30	B	AX	0	F., wealthy politician of Mexico; M., family theatrical; child, "An excellent little actress even now."
106	G	7	6-25-30	B	X	0	F., H.S. English teacher; social science supervisor.
107	G	7	6-25-30	C	A	0	M., teacher; child unwanted, disliked.
108*	B	5	5-15-30	C	N	Y	"An indolent, widely-traveled young man of less than 4 of age."
109	B	7	6-25-30	C	N	Y	Parents slipshod Idealists; F., college graduate, 3rd rate salesman; M., stenog.
110	B	5	5-15-30	E	A	Y	Overshadowed by "managing mamma."
111*	B	5	5-15-30	A	AX	Y	Much initiative; sunny disposition.

Teachers' Supplementary Reports (Continued)

Sub- ject	Sex	School	Date Rated	Number Interest	Temper- ament	Family Member	Remarks
112*	B	5	5-15-30	C	AX	Y	Full of enthusiasm but somewhat negative; bi-lingual difficulty, learning to talk English. Sister, No. 38* above. Parents coll. grads.
113*	B	5	5-15-30	B	AX	Y	Truly spoiled by whole household; has "risen above" his surroundings; F., physician, adores child.
114	G	7	6-25-30	B	X	Y	M., kindergarten teacher. F., in business.
115*	B	7	6-25-30	C	A	Y	M., teacher, ill for months. F., sailor - deserted family.
116*	B	5	5-15-30	C	AX	E	Overshadowed by continual illness; recently appears stronger and more active.
117	B	5	5-15-30				
118	B	5	5-15-30				
119	B	7	6-25-30	C	X	O	F., head, printing dept. in high school; M., cross-eyed, inferiority complex on this account.
120*	G	5	5-15-30	B	AX	O	Only child in N.S. use "baby talk." Otherwise normal. No nurse.
121	B	5	5-15-30	C	A	O	Spolled. Less so as months go by. No nurse.
122	B	7	6-25-30	C	N	Y	Well-adjusted family. F., mechanic; M., clerk.
123	B	7	6-25-30	C	X	O	M., physician; F., clerk.
124	B	7	6-25-30	D	X	O	Babyish. M., vice-principal of H.S. F., salesman.
125	B	5	5-15-30	E	N	Y	"Still a bit of a baby." M., very youthful, interested in family. Older brother - No. 64* above.

Schools reporting:

- No. 1 - Kindergarten, Roosevelt School, Santa Monica, Calif.
Date of Report: 5-13-30; Teacher, Clarissa R. Bacon
- No. 2 - Kindergarten, Alexandria Street School, Los Angeles,
Calif.
Date of Report: 6-10-30; Teacher, Mary K. Adams
- No. 3 - Kindergarten, University Elementary School, U.C.L.A.
Date of Report: 5-25-30; Teacher, Jane Bernhardt
- No. 4 - Nursery School, University Elementary School, U.C.L.A.
Date of Report: 5-29-30; Teacher, Barbara Greenwood
- No. 5 - Nursery School, Progressive School, Hollywood, Calif.
Date of Report: 5-15-30; Teacher, Margaret M. Roberts
- No. 6 - Kindergarten, Progressive School, Hollywood, Calif.
Date of Report: 5-20-30; Teacher, Ruth Ekdale
- No. 7 - Nursery School, Normandie Street School, Los Angeles,
Calif.
Date of Report: 6-25-30; Teacher, Helena Gambril

